

LENS and MIRROR NOTES

We begin with the fundamental equation for both (thin) lenses and mirrors; this can be derived in a variety of ways, some complicated, some just using geometry:

$$\frac{1}{s_{object}} + \frac{1}{s_{image}} = \frac{1}{f} \quad \Rightarrow \quad s_{image} = \frac{f s_{object}}{s_{object} - f} = \frac{f}{1 - \frac{f}{s_{object}}} \quad (1.1)$$

Here the symbol s is used for a distance; this is common notation in physics texts. The focal length is f . The distances are measured from the lens or mirror, with sign conventions we will discuss below, along with how we find f (for now assume it is given). We will also need the magnification equation, which is

$$m = -\frac{s_{image}}{s_{object}} = \frac{f}{f - s_{object}} = \frac{1}{1 - \frac{s_{object}}{f}} \quad (1.2)$$

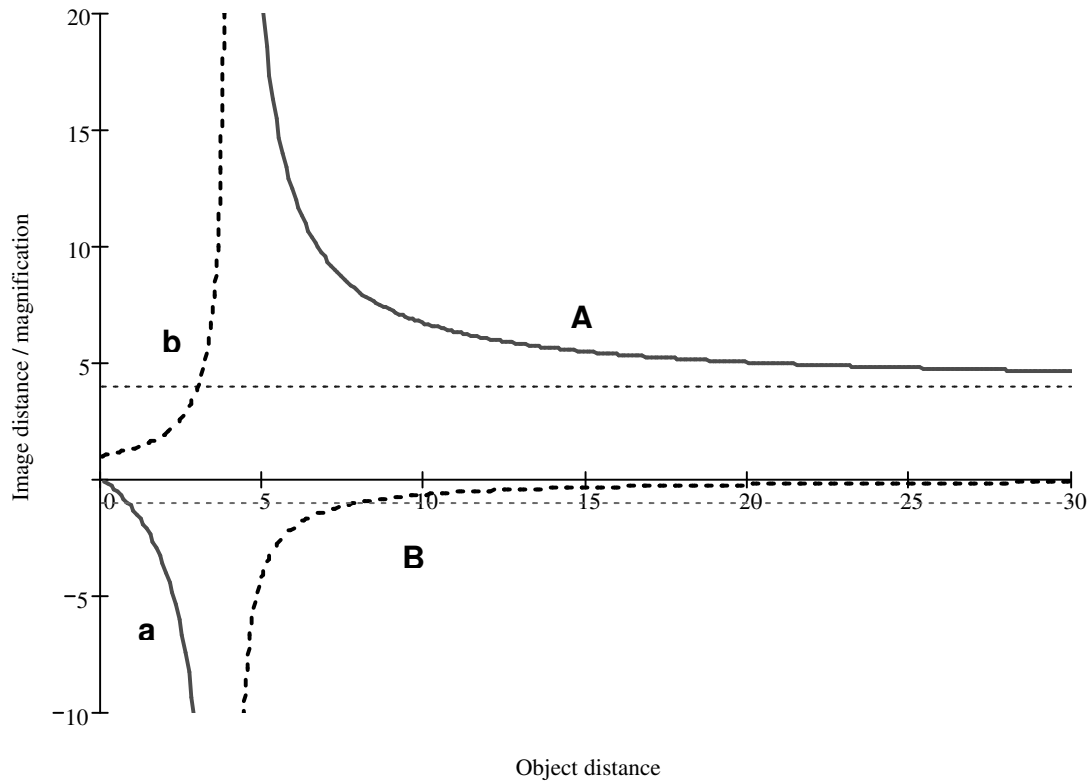


Figure 1. Graphs of the image location (**A** and **a**) and magnification (**B** and **b**) as a function of the object distance from the lens or mirror, using Eqs(1.1) and (1.2).

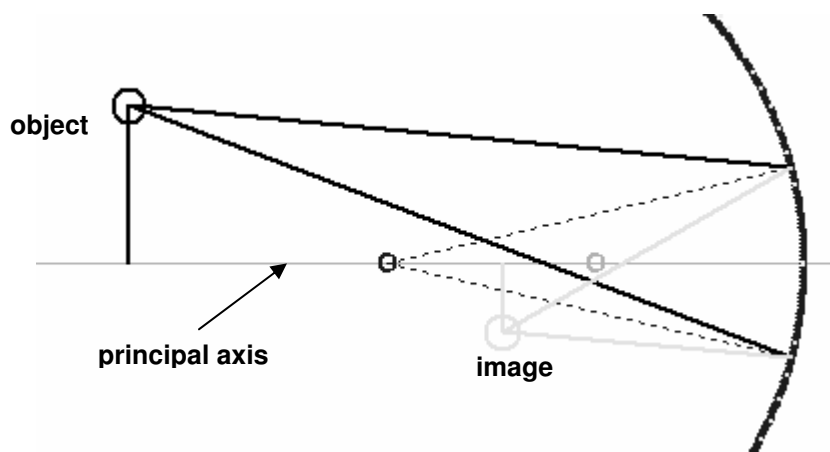
In this example, the focal length is 4, and we see that the image location graph **A** approaches this value as the object distance increases. On the other hand, as the object distance decreases toward f , the image distance increases until it is undefined when the object is exactly at f . Then, as the object distance continues to decrease toward zero, the image location is shown in graph **a**, and it is negative, so the image is virtual in this region.

The magnification graph **B** is negative whenever the object distance is greater than f . This means that any real image (i.e., when the image distance is positive) is inverted. Notice that, as the object distance decreases toward f , the magnification increases, although the image is still inverted. At exactly $2f$, the magnification is -1 . Between $2f$ and f the image is larger and still inverted. In region **b** the image is now virtual, right-side-up, and magnified, until at zero object distance the magnification is again unity.

We can summarize this as follows: Region **a**, virtual image; Region **b**, image upright, larger; Region **A**, real image; Region **B**, image inverted, larger if object distance is less than $2f$, else equal (at $2f$) or smaller.

In short, negative image location means virtual image, and negative magnification means inverted image.

The equations above are based on the *paraxial* assumption. This means that “all rays that diverge from the object make a small angle with the principal axis.” In this sketch, the *principal axis* is the horizontal axis through the center of the mirror.



With this assumption, the math simplifies considerably, including the use of small-angle trig function approximations (similar to that used in the linearized pendulum analysis). We can then show that the focal lengths of a spherical mirror and a thin spherical lens will be:

$$f_{\text{mirror}} = \frac{R}{2} \qquad \frac{1}{f_{\text{lens}}} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \qquad (1.3)$$

Note that the spherical lens has two radii, since the lens can be thought of as the intersection of two spheres (or circles, in 2D). If one side of the lens is flat (a plane) then its corresponding radius is infinite. The index of refraction n is for the lens material, and we assume that the lens is in air (so its $n = 1$). If the lens isn't "thin" then a correction term is added to this expression. Combining the lens part of Eq(1.3) with Eq(1.1) gives what is called the "lensmaker's equation."

Now we need to get organized as to the conventions for measuring the several quantities. These vary from one text to another, but the overall results are not ambiguous, since the light rays do not know what measurement convention we are using, and they will do what they are supposed to do.

light travels from left to right toward a lens or mirror

s-object	positive if in front of (to the left of) a mirror or lens. negative if in back of (to the right of) a lens (no meaning for mirror) [this is positive for practical problems]	
s-image	positive if the image is in front of (to the left of) a mirror negative if the image is apparently behind (to the right of) a mirror positive if the image is in back of (to the right of) a lens negative if the image is apparently in front of (to the left of) a lens	real virtual real virtual
f, R	for mirrors; both positive if center of curvature is in front of mirror “ “ “ “ negative “ “ “ “ back “	concave convex
f-lens	positive for converging lens negative for diverging lens	convex concave
R_1, R_2	positive if the center of curvature is in back of (to the right of) the lens negative “ “ “ “ front “ “ left “ [note that for the usual convex lens, one R will be positive, the other negative]	
m	positive if the image is upright negative “ “ inverted less than unity, image is smaller more “ “ larger	

Note that if an image is real it is inverted. This follows from these conventions and Eq (1.2), since the image distance will be positive. Similarly, if an image is virtual, it is upright. Eq (1.1) also reveals that, if an object is at a (positive) infinite distance, then the image will be at the focal distance f . However, Eq(1.2) shows that when this happens, the image has zero magnification (it is a point). Hence, “focal point.”

We can use Eq(1.1) and (1.2) to discover some interesting things about a convex mirror. According to the conventions, the focal length f of a convex mirror is negative. In Eq(1.1) this means that the image distance must be negative, so that the image is always virtual. In Eq(1.2) we see that a negative f gives a positive m , so that the image is upright. Also in Eq(1.2), since the denominator is greater than unity, the image must be smaller than the object.

Finally, we consider the following table of object and image characteristics for a concave mirror. It can be demonstrated that the same results hold for a thin spherical lens. Essentially, with the appropriate choice of parameters, a concave mirror equals a convex (converging) lens, and a convex mirror equals a concave (diverging) lens.

Location	Type	Location	Orientation	Size
$\infty > s\text{-object} > 2f$	real	$f < s\text{-image} < 2f$	inverted	smaller
$s\text{-object} = 2f$	real	$s\text{-image} = 2f$	inverted	equal
$f < s\text{-object} < 2f$	real	$\infty > s\text{-image} > 2f$	inverted	larger
$s\text{-object} = f$	undefined	undefined	undefined	undefined
$s\text{-object} < f$	virtual	$s\text{-image} > s\text{-object}$	upright	larger