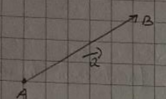
VEKTORI

> Vektor je određen ako mu znamo <u>duljinu</u>, <u>smjer</u>, <u>orjentaciju</u>

> Dva su vektora jednaka ako se podudaraju po duljini, smjeru i orjentaciji

> <u>duljina vektora</u> $\vec{a} = \overrightarrow{AB}$ udaljenost između njegove početne i završne točke oznaka

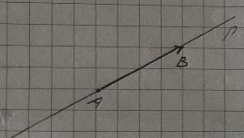
$|\vec{a}| = |\overrightarrow{AB}| = |AB|$ → udaljenost dviju točaka



završna točka

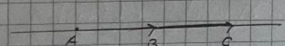
početna točka

> <u>smjer vektora</u> - određen je pravcem na kojem leži vektor



p - pravac nositelj vektora $\overrightarrow{AB}$

> orjentacija vektora



$\overrightarrow{AB}$ i $\overrightarrow{BC}$ iste su orjentacije

$\overrightarrow{BA}$ i $\overrightarrow{CB}$ suprotne su orjentacije

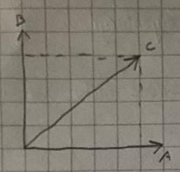
<u>vektor</u> $\overrightarrow{OT}$ nazivamo radijus vektora točke T

<u>suprotni vektori</u> - imaju istu duljinu i smjer, a suprotnu orjentaciju oznaka $\vec{a}$

zbrajanje i oduzimanje vektora → pravilo trokuta

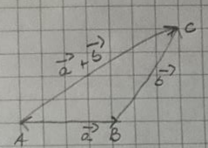
→ pravilo paralelograma

- ako vektori imaju isti početak zbrajamo ih po pravilu paralelograma



$$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$$

- ako se završetak prvog vektora podudara s početkom drugog (nadovezani ili ulančani vektori) zbrajamo po pravilu trokuta



množenje vektora a skalarom λ 16. str.

- kolinearni vektori su linearno zavisni. Naime ako je $\alpha_1 \vec{a}_1 + \alpha_2 a_2 = \vec{0}$

$\lambda \neq 0$

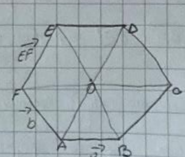
$$\Rightarrow \vec{a}_1 = \frac{\alpha_2}{\alpha_1} \cdot \vec{a}_2 = k\vec{a}_2$$

Pr.

1)

Šesterokut ABCDEF. Ako je $\vec{AB} = \vec{c}$ $\vec{AF} = \vec{b}$

pomoću $\vec{a}$ i $\vec{b}$ izraziti $\vec{CE}, \vec{EF}, \vec{BD}, \vec{BE}$



vektori $\vec{OC}$ i $\vec{AB}$ su jednaki, $\vec{AF} = \vec{OE} = \vec{CD} = \vec{b}$

$\vec{DE} = \vec{AB} = \vec{a}$

$\vec{CE} = \vec{CD} + \vec{DE}$

$\vec{CE} = \vec{b} - \vec{a}$

$\vec{EF} = \vec{EO} + \vec{OF} = -\vec{b} - \vec{a}$

$\vec{BD} = \vec{BE} + \vec{ED} = 2\vec{b} + \vec{a}$

$\vec{BE} = 2\vec{b}$

Dane su točke $A(-3,1)$ $B(-1,-2)$ $C(2,3)$. Odredi vektore $\vec{AB}$, $\vec{BC}$ i $\vec{AC}$

$\vec{AB} = (x_B - x_1)\vec{i} + (y_B - y_1)\vec{j}$

$A(-3,1)$ $B(-1,-2)$
x_A y_A x_B y_B

$\vec{AB} = (-1-1-3)\vec{i} + (-2-1)\vec{j} = 2\vec{i} - 3j$

$\vec{BC} = (x_C - x_B)\vec{i} + (y_C + y_B)\vec{j} = (2-(-1))\vec{i} + (3-(-2)) = 3\vec{i} + 5\vec{j}$

$B(-1,-2)$ $C(2,3)$
x_B y_B x_C y_C

$\vec{AC} = (x_C - x_A)\vec{i} + (y_C - y_A)\vec{j} = (2-(-3))\vec{i} + (3-1)\vec{j} = 5\vec{i} + 2\vec{j}$

$A(-3,1)$ $C(2,3)$
x_A y_A x_C y_C

Pr.

1. Vektor $\vec{c} = \vec{i} + 5\vec{j}$ prikaži kao linearnu kombinaciju vektora $\vec{a} = \vec{i} - \vec{j}$ i $\vec{b} = 2\vec{i} + \vec{j}$

$\vec{c} = \alpha_1 \vec{a} + \alpha_2 \vec{b}$

$\vec{i} + 5\vec{j} = \alpha_1(\vec{i} - \vec{j}) + \alpha_2(2\vec{i} + \vec{j})$

$\vec{i} + 5\vec{j} = \alpha_1 \vec{i} - \alpha_1 \vec{j} + 2\alpha_2 \vec{i} + \alpha_2 j$

$\vec{i} + 5j = i^2(\alpha_1 + 2\alpha_2) + \vec{j}(-\alpha_1 + \alpha_2)$

$1 = \alpha_1 + 2\alpha_2$

$5 = -\alpha_1 + \alpha_2$

$6 = 3\alpha_2 \quad / :3$

$\boxed{\alpha_2 = 2}$

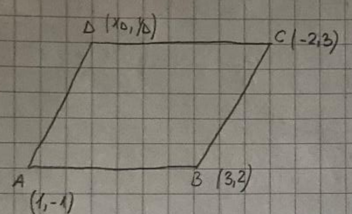
$1 = \alpha_1 + 2\cdot 2$

$1 = \alpha_1 + 4 \rightarrow \alpha_1 = 3$

$\vec{c} = -3\vec{a} + 2\vec{b}$

1. Ako su A, B, C - tri uzastopna vrha paralelograma ABCD, odredi vrh D

$A(1,-1) \quad B(3,2) \quad C(-2,3)$



$\overrightarrow{AD} = \overrightarrow{BC}$

$\overrightarrow{AD} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$

$\overrightarrow{AD} = (x_D - 1)\vec{i} + (y_D + 1)\vec{j}$ //

$\overrightarrow{BD} = (-2-3)\vec{i} + (3-2)\vec{j}$

$\overrightarrow{BD} = -5\vec{i} + 1\vec{j}$ //

$\overrightarrow{AD} = \overrightarrow{BC}$

$(x_D - 1)\vec{i} + (y_D + 1)\vec{j} = -5\vec{i} + 1\vec{j}$

$x_D - 1 = -5$

$x_D = 1 - 5$

$x_D = -4$

$y_D + 1 = 1$

$y_D = 1 - 1$

$y_D = 0$

$D(-4, 0)$

Pr. 3 Vektor $\overrightarrow{BD}$ prikaži kao linearnu kombinaciju vektora $\overrightarrow{AB}$ i $\overrightarrow{BC}$ ako je $A(-1,2)$, $B(1,1)$, $C(2,-3)$

$D(-7,-2)$

$\overrightarrow{BD} = (-7,-1)\vec{i} + (-2,-1)\vec{j} = -8\vec{i} - 3\vec{j}$

$\overrightarrow{AB} = 2\vec{i} - \vec{j}$

$\overrightarrow{BC} = \vec{i} - 4\vec{j}$

$\overrightarrow{BD} = \alpha_1 \overrightarrow{AB} + \alpha_2 \overrightarrow{BC}$

$-8\vec{i} - 3\vec{j} = \alpha_1(2\vec{i} - \vec{j}) + \alpha_2(\vec{i} - 4\vec{j})$

$-8\vec{i} - 3\vec{j} = 2\alpha_1\vec{i} - \alpha_1\vec{j} + \alpha_2\vec{i} - 4\alpha_2\vec{j}$

$-8 = 2\alpha_1 + 2\alpha_2$

$-3 = -\alpha_1 - 4\alpha_2 \quad / \cdot 2$

$-8 = 2\alpha_1 + \alpha_2$

$-6 = -2\alpha_1 - 8\alpha_2$

$-14 = -7\alpha_2 \quad / : (-7)$

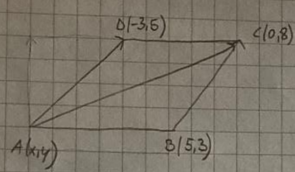
$\alpha_2 = 2$

$-8 = 2\alpha_1 + 2$

$2\alpha_1 = -10 \quad / : 2$

$\alpha_1 = -5$

Pr. 3, Kolika je duljina dijagonale $\vec{AC}$ paralelograma ABCD kojemu su točke $B(5,3)$, $C(0,8)$, $D(-3,5)$



$\vec{AD} = \vec{BC}$

$(-3-x)\vec{i} + (5-y)\vec{j} = (0-5)\vec{i} + (8-3)\vec{j}$

$(-3-x)\vec{i} + (5-y)\vec{j} = -5\vec{i} + 5\vec{j}$

$-3-x = -5 \Rightarrow -x = -5+3$

$-x = -2$

$x = 2$

$5-y = 5$

$y = 0 \Rightarrow A(2,0)$

$A(2,0)$

$\vec{AC} = (0-2)\vec{i} + (8-0)\vec{j}$

$\vec{AC} = -2\vec{i} + 8\vec{j}$

$|\vec{AC}| = \sqrt{4+64} = \sqrt{68}$ //

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