

En los inciso i), ii), iii) y iv) siguientes
a) determine la matriz de transición de B_1 a B_2 .

$$i) B_1 = \{(1, 3), (-2, -2)\}, B_2 = \{(-12, 0), (-4, 4)\}, [u]_{B_2} = (-1, 3)$$

Solución:

$$\begin{aligned} [B_2/B_1] &= \left[\begin{array}{cc|cc} -12 & -4 & 1 & -2 \\ 0 & 4 & 3 & -2 \end{array} \right] [f_2 + f_1 \rightarrow f_1] \sim \\ & \left[\begin{array}{cc|cc} -12 & 0 & 4 & -4 \\ 0 & 4 & 3 & -2 \end{array} \right] \left[\begin{array}{l} -\frac{1}{2}f_1 \rightarrow f_1 \\ \frac{1}{4}f_2 \rightarrow f_2 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{3}{4} & -\frac{1}{2} \end{array} \right] \end{aligned}$$

Por lo tanto

$$P_{B_1 \rightarrow B_2} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{3}{4} & -\frac{1}{2} \end{bmatrix}$$

$$ii) B_1 = \{(2, -2), (6, 3)\}, B_2 = \{(1, 1), (32, 31)\}, [u]_{B_2} = (2, -1)$$

Solución:

$$\begin{aligned} [B_2/B_1] &= \left[\begin{array}{cc|cc} 1 & 32 & 2 & 6 \\ 1 & 31 & -2 & 3 \end{array} \right] [-f_1 + f_2 \rightarrow f_2] \sim \left[\begin{array}{cc|cc} 1 & 32 & 2 & 6 \\ 0 & -1 & -4 & -3 \end{array} \right] [32f_2 + f_1 \rightarrow f_1] \sim \\ & \left[\begin{array}{cc|cc} 1 & 0 & -126 & -90 \\ 0 & -1 & -4 & -3 \end{array} \right] [-f_2 \rightarrow f_2] \sim \left[\begin{array}{cc|cc} 1 & 0 & -126 & -90 \\ 0 & 1 & 4 & 3 \end{array} \right] \end{aligned}$$

Por lo tanto

$$P_{B_1 \rightarrow B_2} = \begin{bmatrix} -126 & -90 \\ 4 & 3 \end{bmatrix}$$

$$iii) B_1 = \{(4, 2, -4), (6, -5, -6), (2, -1, 8)\}, B_2 = \{(1, 0, 4), (4, 2, 8), (2, 5, -2)\}, [u]_{B_2} = (1, -1, 2)$$

Solución:

$$\begin{aligned} [B_2/B_1] &= \left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 4 & 6 & 2 \\ 0 & 2 & 5 & 2 & -5 & -1 \\ 4 & 8 & -2 & -4 & -6 & -3 \end{array} \right] [-4f_1 + f_3 \rightarrow f_3] \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 4 & 6 & 2 \\ 0 & 2 & 5 & 2 & -5 & -1 \\ 0 & -8 & -10 & -20 & -30 & -16 \end{array} \right] [\frac{1}{2}f_2 \rightarrow f_2] \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 4 & 6 & 2 \\ 0 & 1 & \frac{5}{2} & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -8 & -10 & -20 & -30 & -16 \end{array} \right] \left[\begin{array}{l} 8f_2 + f_3 \rightarrow f_3 \\ -4f_2 + f_1 \rightarrow f_1 \end{array} \right] \sim \end{aligned}$$

$$\begin{aligned} & \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 14 & 4 \\ 0 & 1 & \frac{5}{2} & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 10 & -18 & -10 & -20 \end{array} \right] \left[\begin{array}{l} \frac{1}{10}f_3 \rightarrow f_3 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 0 & 14 & 4 \\ 0 & 1 & \frac{5}{2} & 1 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{9}{5} & -1 & -2 \end{array} \right] \left[\begin{array}{l} 8f_3 + f_1 \rightarrow f_1 \\ -\frac{5}{2}f_3 + f_2 \rightarrow f_2 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{72}{5} & -22 & -12 \\ 0 & 1 & 0 & \frac{11}{2} & 5 & \frac{4}{2} \\ 0 & 0 & 1 & -\frac{9}{5} & -1 & -2 \end{array} \right] \end{aligned}$$

Por lo tanto

$$P_{B_1 \rightarrow B_2} = \begin{bmatrix} -\frac{72}{5} & -22 & -12 \\ \frac{11}{2} & 5 & \frac{4}{2} \\ -\frac{9}{5} & -1 & -2 \end{bmatrix}$$

iv) $B_1 = \{(1, 3, 4), (2, -5, 2), (-4, 2, -6)\}$, $B_2 = \{(1, 2, -2), (4, 1, -4), (-2, 5, 8)\}$
Solución:

$$\begin{aligned} [B_2/B_1] &= \left[\begin{array}{ccc|ccc} 1 & 4 & -2 & 1 & 2 & -4 \\ 2 & 1 & 5 & 3 & -5 & 2 \\ -2 & -4 & 8 & 4 & 2 & -6 \end{array} \right] \left[\begin{array}{l} -2f_1 + f_2 \rightarrow f_2 \\ 2f_1 + f_3 \rightarrow f_3 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 4 & -2 & 1 & 2 & -4 \\ 0 & -7 & 9 & 1 & -9 & 10 \\ 0 & 4 & 4 & 6 & 6 & -14 \end{array} \right] \left[\begin{array}{l} -\frac{1}{7}f_2 \rightarrow f_2 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 4 & -2 & 1 & 2 & -4 \\ 0 & 1 & -\frac{9}{7} & -\frac{1}{7} & \frac{9}{7} & \frac{10}{7} \\ 0 & 4 & 4 & 6 & 6 & -14 \end{array} \right] \left[\begin{array}{l} -4f_2 + f_1 \rightarrow f_1 \\ -4f_2 + f_3 \rightarrow f_3 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{22}{7} & \frac{11}{7} & -\frac{22}{7} & \frac{12}{7} \\ 0 & 1 & -\frac{9}{7} & -\frac{1}{7} & \frac{9}{7} & \frac{10}{7} \\ 0 & 0 & \frac{64}{7} & \frac{46}{7} & \frac{6}{7} & -\frac{58}{7} \end{array} \right] \left[\begin{array}{l} \frac{7}{64}f_3 \rightarrow f_3 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{22}{7} & \frac{11}{7} & -\frac{22}{7} & \frac{12}{7} \\ 0 & 1 & -\frac{9}{7} & -\frac{1}{7} & \frac{9}{7} & \frac{10}{7} \\ 0 & 0 & 1 & \frac{23}{32} & \frac{3}{32} & -\frac{29}{32} \end{array} \right] \left[\begin{array}{l} -\frac{22}{7}f_3 + f_1 \rightarrow f_1 \\ -\frac{9}{7}f_3 + f_2 \rightarrow f_2 \\ \sim \end{array} \right] \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -\frac{11}{16} & -\frac{55}{16} & \frac{73}{16} \\ 0 & 1 & 0 & -\frac{25}{32} & \frac{45}{32} & -\frac{59}{32} \\ 0 & 0 & 1 & \frac{23}{32} & \frac{3}{32} & -\frac{29}{32} \end{array} \right] \end{aligned}$$

Por lo tanto

$$P_{B_1 \rightarrow B_2} = \begin{bmatrix} -\frac{11}{16} & -\frac{55}{16} & \frac{73}{16} \\ -\frac{25}{32} & \frac{45}{32} & -\frac{59}{32} \\ \frac{23}{32} & \frac{3}{32} & -\frac{29}{32} \end{bmatrix}$$