Chapter 14

Coordinate geometry

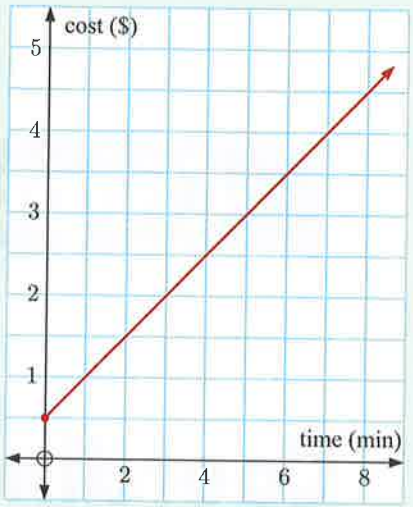
Contents:

A The Cartesian plane
B Linear relationships
C Gradient
D Axes intercepts
E The equation of a line
F Graphing lines from equations
G Vertical and horizontal lines
H Finding equations from graphs



OPENING PROBLEM

When Tiffany makes a mobile phone call, the *cost* of the call depends on the *time* that she spends on the call. The relationship between the cost and the time of the call is shown on the graph alongside.



Things to think about:

a Is the relationship between *cost* and *time* linear?

b What is the *connection fee* or fixed cost charged for a very short phone call?

c By how much does the cost of the call increase for each minute spent on the call?

d Can you use this graph to determine:

i the cost of a 3 minute call

ii the length of a call which costs $4?

The graph in the **Opening Problem** is a straight line. We say there is a **linear relationship** between the variables.

HISTORICAL NOTE

Sir Isaac Newton is recognised as one of the great mathematicians of all time. His achievements are remarkable considering mathematics, physics, and astronomy were enjoyable pastimes after his main interests of chemistry, theology, and alchemy.





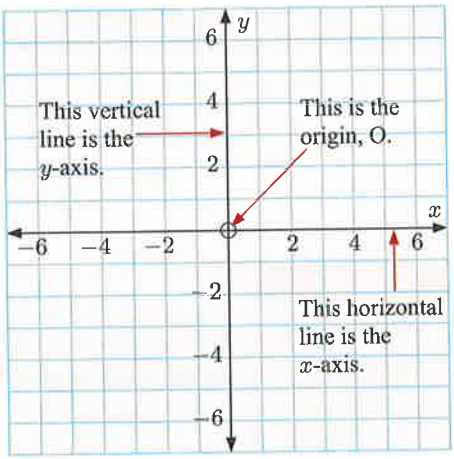
Despite his obvious abilities, there is a story from Newton's childhood which indicates that even the greatest thinkers can find silly solutions for simple problems. He was asked to go out and cut a hole in the bottom of the barn door for the cats to go in and out. He decided to cut two holes: one for the cat and a smaller one for the kittens.

After completing school, Newton was initially made to work on a farm, but when his uncle discovered his enthusiasm for mathematics it was decided that he should attend Cambridge University.

Newton's contribution to coordinate geometry included the introduction of negative values for coordinates. In his *Method of Fluxions*, Newton suggested eight new types of coordinate systems, one of which we know today as **polar coordinates**.

A THE CARTESIAN PLANE

The number grid alongside is a **Cartesian plane**, named after **René Descartes**. The numbers or **coordinates** on it allow us to locate the exact position of any point on the plane.

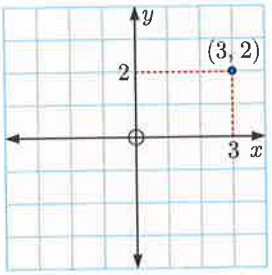


We start with a point of reference O called the **origin**. Through it we draw two fixed lines which are perpendicular to each other. They are a horizontal line called the ***x*-axis**, and a vertical line called the ***y*-axis**.

The ***x*-axis** is an ordinary number line with positive numbers to the right of O and negative numbers to the left of O. The ***y*-axis** has positive numbers above O and negative numbers below O.

PLOTTING POINTS ON THE CARTESIAN PLANE

To specify the position of a point on the number plane, we use an **ordered pair** of numbers in the form (x, y).



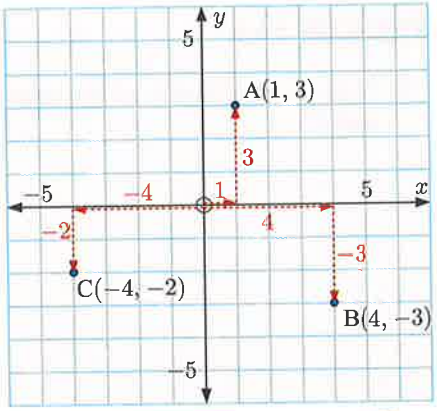
For example, on the grid alongside we see the point described by the ordered pair (3, 2). We say that the point has **coordinates** (3, 2).

To help us identify particular points, we often refer to them using a capital letter. For example, consider the points A(1, 3), B(4, −3), and C(−4, −2).

To plot the point A(1, 3):

- start at the origin O
- move right along the x-axis 1 unit
- then move upwards 3 units.

We say that 1 is the ***x*-coordinate** of A
and 3 is the ***y*-coordinate** of A.



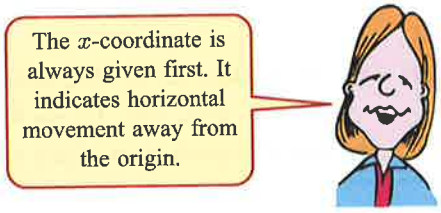
To plot the point B(4, −3):

- start at the origin O
- move right along the x-axis 4 units
- then move downwards 3 units.

To plot the point C(−4, −2):

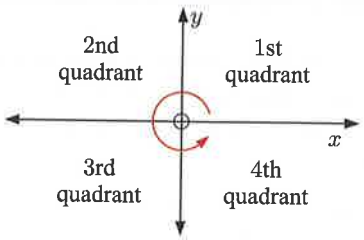
- start at the origin O
- move left along the x-axis 4 units
- then move downwards 2 units.

DEMO



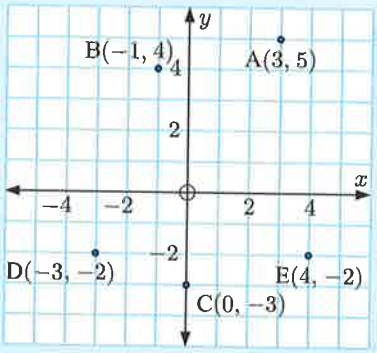
QUADRANTS

The x and y-axes divide the Cartesian plane into four regions referred to as **quadrants**. These quadrants are numbered in an **anti-clockwise direction** as shown:



Example 1 **Self Tutor**

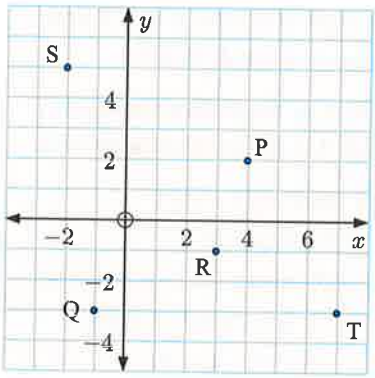
Plot the points A(3, 5), B(−1, 4), C(0, −3), D(−3, −2), and E(4, −2) on the same set of axes.





EXERCISE 14A

1 State the coordinates of the points P, Q, R, S, and T:

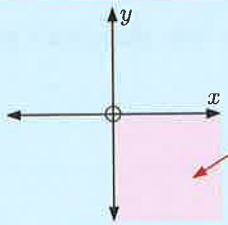


2 **a** On the same set of axes, plot the points:
A(2, 5), B(4, −2), C(−5, 0), D(−1, −4), E(−2, 3), F(0, 3), G(5, 1), and H(−5, −1).

b State the quadrant in which each point in **a** lies.

Example 2 **Self Tutor**

On a Cartesian plane, show all the points with positive x-coordinate and negative y-coordinate.



This shaded region contains all points where x is positive and y is negative.
The points on the axes are not included.



3 On different sets of axes, show all the points with:

a x-coordinate equal to -1 **b** y-coordinate equal to 3

c x-coordinate equal to 0 **d** y-coordinate equal to 0

e negative x-coordinate **f** positive y-coordinate

g negative x and y-coordinates **h** negative x-coordinate and positive y-coordinate.

4 State the quadrants in which I would find points whose coordinates have:

a the same sign **b** different signs.

B LINEAR RELATIONSHIPS

Consider the pattern:



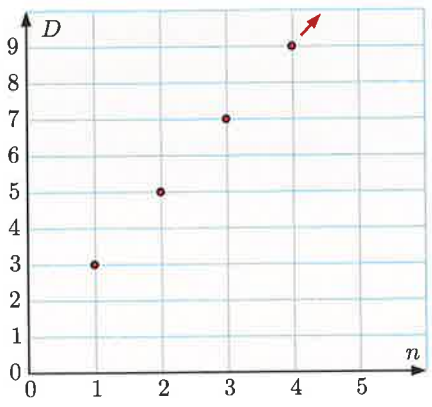
We can construct a **table of values** which connects the diagram number n to the number of dots D. To go from one diagram to the next we need to add *two more* dots.

n	1	2	3	4
D	3	5	7	9
	+2	+2	+2	

The **equation** which connects n and D in this case is $D = 2n + 1$.

The number of dots D *depends* on the diagram number n. We say that n is the **independent variable** and D is the **dependent variable**.

We can plot the relationship between D and n on a number plane. We place the independent variable n on the horizontal axis, and the dependent variable D on the vertical axis.



This is an example of a **linear relationship** because the points lie in a straight line.

In this case the values in between the points are meaningless. For example, we cannot have a $2\frac{1}{2}$th diagram. We therefore do not connect the points with a straight line.

However, there are many situations where it *is* sensible to connect the points with a straight line. In these cases, we can use the line to answer questions involving values between the given points.

Example 3 Self Tutor

Max has 10 litres of fuel left in his car's petrol tank. When he fills it at the petrol station, petrol runs into the tank at 15 litres per minute. The petrol tank can hold 70 litres.

a Identify the independent and dependent variables.

b Make a table of values for the number of litres L of petrol in the tank after time t minutes, and plot the graph of L against t.

c Is the relationship between L and t linear?

d Is it sensible to join the points graphed with a straight line?

e For every time increase of 1 minute, what is the change in L?

f Find the number of litres of petrol in the tank after 1.5 minutes.

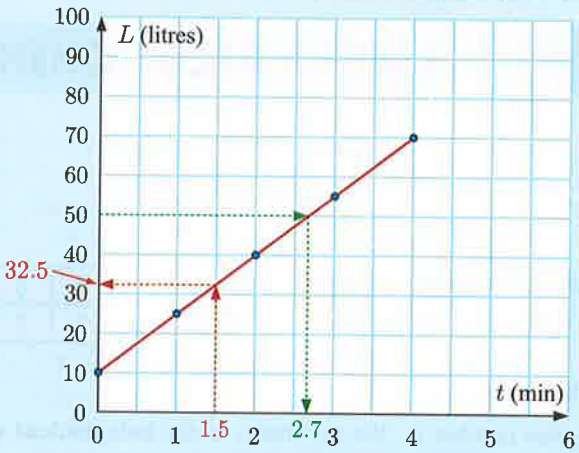
g At what time will there be 50 litres of petrol in the tank?

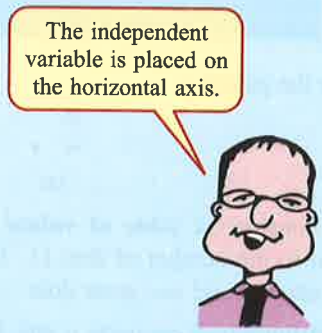
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a The *number of litres of petrol* in the tank depends on the *time* it has been filling.
$\therefore$ *time* is the independent variable and the *number of litres of petrol* is the dependent variable.

b Each minute the tank is filling adds another 15 litres of petrol.

t (min)	0	1	2	3	4
L (litres)	10	25	40	55	70





c The points lie in a straight line so the relationship is linear.

d Yes, as Max could add petrol for 2.5 minutes, say, or put 56 litres of petrol in the tank.

e For every time increase of 1 minute, L increases by 15 litres.

f After 1.5 minutes there are 32.5 litres of petrol in the tank.

g There are 50 litres of petrol in the tank after about 2.7 minutes.

EXERCISE 14B

1 Each week a department store employee receives a basic salary of \$300. In addition, she is paid a bonus of \$20 for each new member she signs up to the store's rewards club. Let I be the employee's income and m be the number of members she signs up to the rewards club.



a What are the independent and dependent variables?

b Construct a table of values for I, for $m = 0, 1, 2, 3, ..., 10$.

c Hence draw a graph of I against m.

d Is the relationship linear?

e Is it sensible to join the points with a straight line? Explain your answer.

f For each new member signed, what is the employee's increase in income?

2 Mangoes can be bought for €3.50 each.

a Copy and complete the table:

Number of mangoes, n	0	1	2	3	4	5	6	7	8
Cost, €C									

b Plot the graph of C against n.

c Identify the independent and dependent variables.

d Is the relationship between C and n linear?

e Is it sensible to join the points with a straight line? Explain your answer.

f For each extra mango bought, what is the change in C?

g Find the cost of 5 mangoes.

h How many mangoes could be bought for €24.50?

3 Simon is filling a large container with sports drink for his football team. It already contains 3 litres when he starts to fill it up using 2 L bottles.



a Make a table of values for the volume of sports drink S in the container after Simon has emptied n bottles of sports drink into it. Consider $n = 0, 1, 2, ..., 8$.

b What are the independent and dependent variables?

c Plot the graph of S against n.

d Is the relationship between S and n linear?

e Is it sensible to join the points graphed with a straight line? Explain your answer.

f For each full bottle of sports drink added, what is the change in S?

g What volume of sports drink is in the container after Simon has emptied $2\frac{1}{2}$ bottles into it?

h How many full bottles must be emptied into the container so it contains 15 L in total?

4 Adrian's cookbook gives the cooking temperatures in degrees Fahrenheit (°F). However, Adrian is only familiar with degrees Celsius (°C).

a Draw a set of axes as shown.

b Identify the independent and dependent variables.

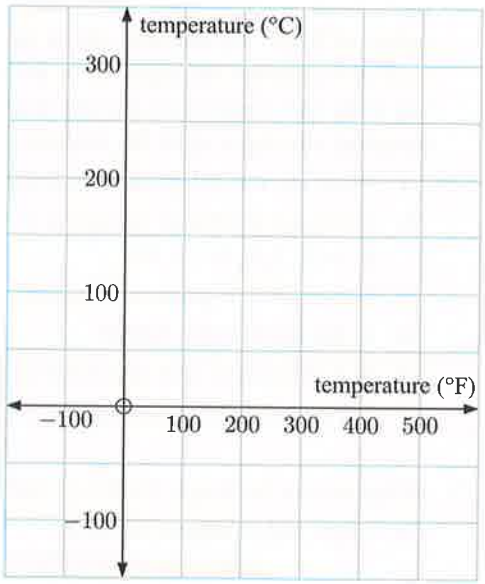
c There is a linear relationship between °F and °C. The boiling point of water is 212 °F or 100 °C. The freezing point of water is 32 °F or 0 °C. Mark these points on your graph and join them with a straight line.

d Find the point where the number of degrees Celsius equals the same number of degrees Fahrenheit. What is the temperature?

e Help Adrian convert these temperatures into °C:

i a cake must be cooked at 350 °F

ii a roast must reach an internal temperature of 145 °F.



f Use your graph to complete the following table:

Temperature in °F	0	180		
Temperature in °C			150	200

PRINTABLE GRID

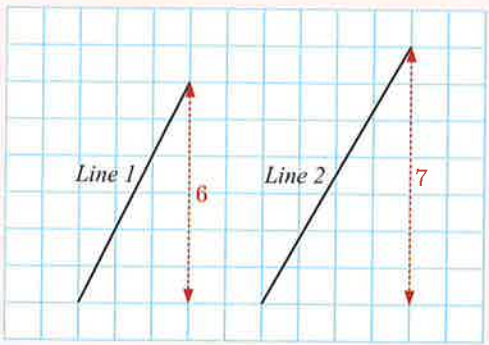
C GRADIENT

DISCUSSION WHICH LINE IS STEEPER?

Look at the two lines alongside.

Justin thinks that *line 2* is steeper, because *line 2* rises 7 units vertically, whereas *line 1* only rises 6 units.

- Is Justin correct? Which line do you think is steepest?
- What does it mean to say that one line is *steeper* than another?
- How can we *measure* the steepness of a line?

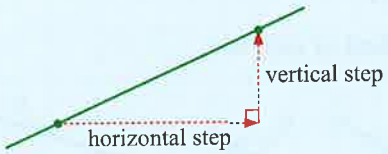


The **gradient** of a line is a measure of its steepness.

In the **Discussion** above, *line 2* rises higher than *line 1*, but *line 1* is *steeper* because it rises at a greater *rate* than *line 2*.

We can calculate the rate at which a line rises or falls by choosing two points on the line, then dividing the vertical step between them by the horizontal step between them.

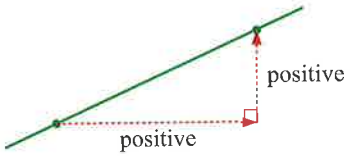
$$\text{gradient} = \frac{\text{vertical step}}{\text{horizontal step}}$$



The gradient of *line 1* is $\frac{6}{3} = 2$. This means that for every 1 unit moved horizontally, the line moves 2 units upwards.

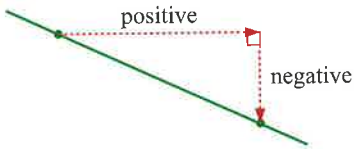
The gradient of *line 2* is $\frac{7}{4} = 1.75$. *Line 1* has a higher gradient than *line 2*, which means that *line 1* is steeper than *line 2*.

For an upwards sloping line, if the horizontal step is positive then the vertical step is positive.



Upward sloping lines have a **positive gradient**.

For a downward sloping line, if the horizontal step is positive then the vertical step is negative.



Downward sloping lines have a **negative gradient**.

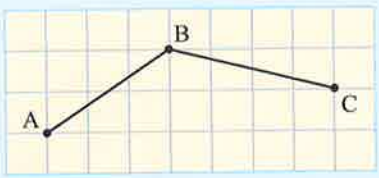
For a **horizontal line**, the vertical step is 0, so the **gradient of a horizontal line is 0**.

For a **vertical line**, the horizontal step is 0, so the **gradient of a vertical line is undefined**.

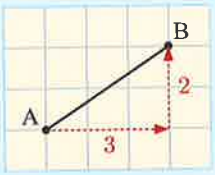
Example 4 Self Tutor

Find the gradient of:

a [AB] **b** [BC]

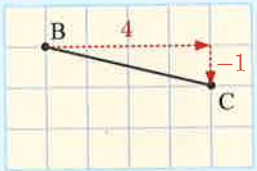


a



$$\text{gradient} = \frac{\text{vertical step}}{\text{horizontal step}}$$
$$= \frac{2}{3}$$

b

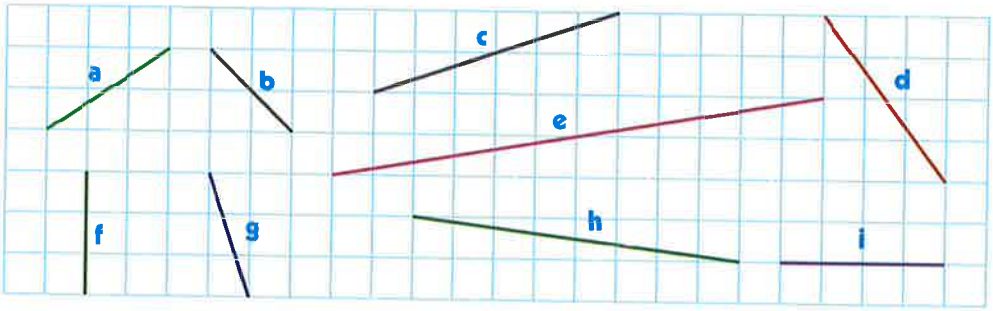


$$\text{gradient} = \frac{\text{vertical step}}{\text{horizontal step}}$$
$$= \frac{-1}{4}$$
$$= -\frac{1}{4}$$



EXERCISE 14C.1

1 State the gradient of each line:



2 On grid paper, draw a line segment with gradient:

a $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $-\frac{1}{4}$ **d** $2 = \frac{2}{1}$ **e** 1 **f** -3

g $\frac{4}{3}$ **h** $-\frac{5}{2}$ **i** 0 **j** $1\frac{1}{5}$ **k** 5 **l** undefined

3 On grid paper, draw a triangle which has:

a all three sides with positive gradients

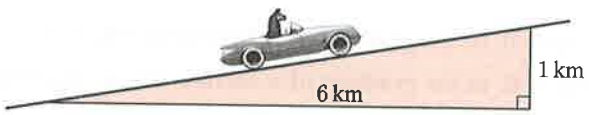
b two sides with positive gradients and one side with negative gradient

c two sides with negative gradients and one side with positive gradient

d all three sides with negative gradients.

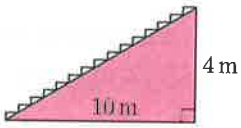
4 Find the gradient of the following:

a



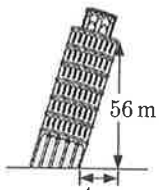
road uphill

b



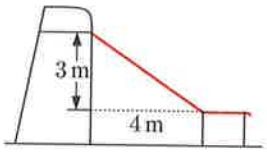
stairs

c



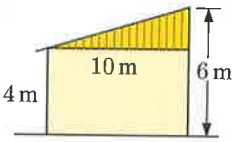
tower

d



slippery-dip

e



barn roof

5 **a** Determine the gradient of:

i [OA] **ii** [OB] **iii** [OC]

iv [OD] **v** [OE] **vi** [OF]

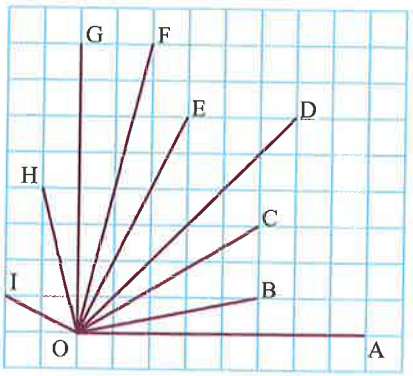
vii [OG] **viii** [OH] **ix** [OI]

b Copy and complete the following statements:

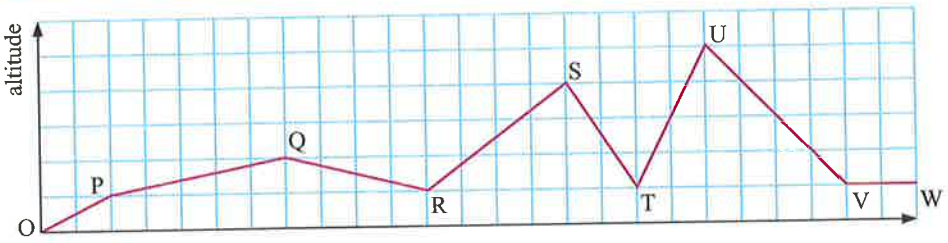
i The gradient of a horizontal line is .....

ii The gradient of a vertical line is .....

iii As line segments become steeper, their gradients .....



6



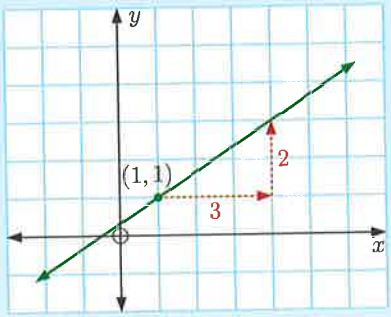
Imagine that you are walking across the countryside from O to W, from left to right. The vertical axis of the grid shows your altitude, or height above your starting point.

a When are you going uphill?
b When are you going downhill?
c Where is the steepest positive gradient?
d Where is the steepest negative gradient?
e Where is the gradient 0?
f Where is the least steep positive gradient?

Example 5 **Self Tutor**

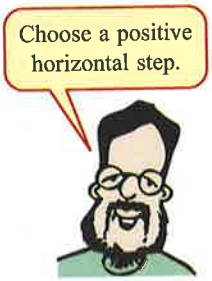
Draw a line through the point $(1, 1)$, with gradient $\frac{2}{3}$.

gradient $= \frac{2}{3}$ ← vertical step
← horizontal step



7 Draw a line through the point:

a $(4, 1)$ with gradient $\frac{3}{2}$
b $(3, -1)$ with gradient $\frac{1}{4}$
c $(-2, 3)$ with gradient $-\frac{1}{5}$
d $(0, 4)$ with gradient $-\frac{2}{3}$
e $(0, -2)$ with gradient 3
f $(-2, -1)$ with gradient -2
g $(0, 6)$ with gradient $-\frac{5}{2}$
h $(0, -3)$ with gradient $-\frac{3}{4}$.



8 By plotting the points on graph paper, find the gradient of the line segment joining:

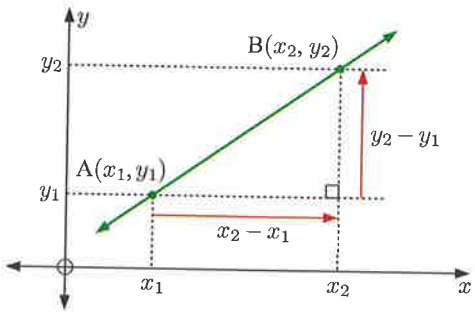
a O(0, 0) and A(2, 6)
b O(0, 0) and B(−4, 2)
c G(0, −1) and H(2, 5)
d K(1, 1) and L(−2, −2)
e M(3, 1) and N(−1, 3)
f P(−2, 4) and Q(2, 0).

THE GRADIENT FORMULA

Although we can find gradients using steps on a diagram, it is often quicker to use a formula.

For points $A(x_1, y_1)$ and $B(x_2, y_2)$, the vertical step is $y_2 - y_1$, and the horizontal step is $x_2 - x_1$.

$\therefore$ the gradient is $\dfrac{y_2 - y_1}{x_2 - x_1}$.



The **gradient** of the line through (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$.

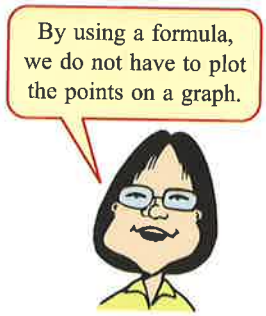
Example 6 Self Tutor

Find the gradient of [PQ] for P(1, 3) and Q(4, −2).

P(1, 3) Q(4, −2)

x_1 y_1 x_2 y_2

$$\text{gradient of [PQ]} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{4 - 1} = \frac{-5}{3} = -\frac{5}{3}$$

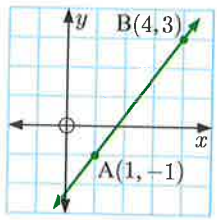


EXERCISE 14C.2

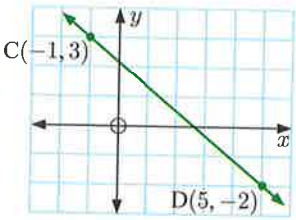
1 For each graph, find the gradient of the line using:

i horizontal and vertical steps **ii** the gradient formula.

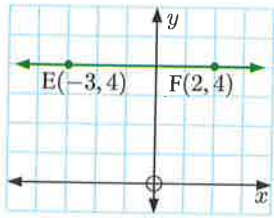
a



b



c



2 Use the gradient formula to find the gradient of the line segment joining:

a O(0, 0) and A(2, 6) **b** O(0, 0) and B(−4, 2) **c** O(0, 0) and C(2, −12)

d O(0, 0) and D(1, −5) **e** E(1, 0) and F(1, 5) **f** G(0, −1) and H(2, −1)

g I(1, 1) and J(3, 3) **h** S(2, 3) and T(−2, −7) **i** J(4, 1) and K(−2, 3)

j A(4, 6) and B(−8, −3). **k** P(1, −3) and Q(−5, −7) **l** M(6, 2) and N(−2, −16).

3 Which method do you think is better to find the gradient of the line joining R(−50, 460) and S(115, 55)? Explain your answer.

4 A$(-2, 3)$, B$(3, 6)$, C$(6, 2)$, and D$(-4, -4)$ are vertices of a quadrilateral. The sides [AB] and [CD] are parallel.

a Plot quadrilateral ABCD on a graph.

b What type of quadrilateral is ABCD?

c Find the gradient of:

i [AB] **ii** [BC] **iii** [CD] **iv** [DA]

d What do you suspect we can say about the gradients of parallel lines?

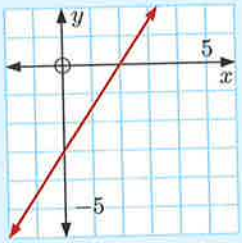
D AXES INTERCEPTS

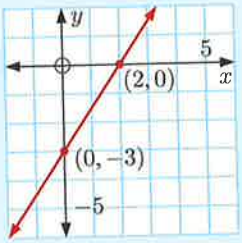
When we are investigating lines on a number plane, it is useful to know where the line cuts the x and y-axes.

The ***x*-intercept** of a line is the x-coordinate of the point where the line cuts the x-axis.
The ***y*-intercept** of a line is the y-coordinate of the point where the line cuts the y-axis.

Example 7 Self Tutor

Find the x and y-intercepts of the line given:





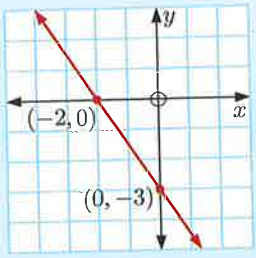
The line cuts the x-axis at $(2, 0)$, so the x-intercept is 2.

The line cuts the y-axis at $(0, -3)$, so the y-intercept is -3.

If the axes intercepts of a line are non-zero, we can quickly sketch the line and find its gradient.

Example 8 Self Tutor

Draw the graph of the line with x-intercept -2 and y-intercept -3.
Find the gradient of the line.



The line passes through $(-2, 0)$ and $(0, -3)$.

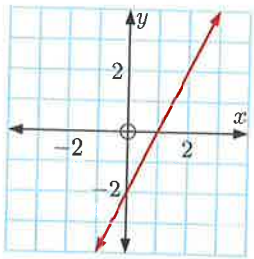
Its gradient is $\dfrac{-3-0}{0-(-2)} = \dfrac{-3}{2}$

$= -\dfrac{3}{2}$

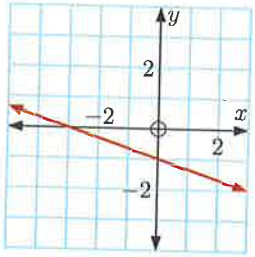
EXERCISE 14D

1 Find the x and y-intercepts of the following lines:

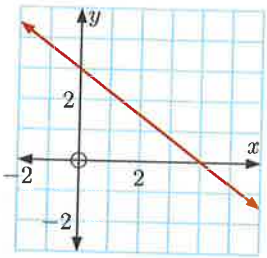
a



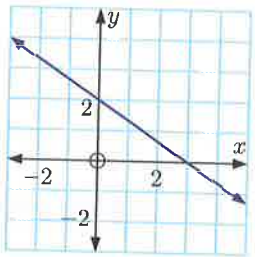
b



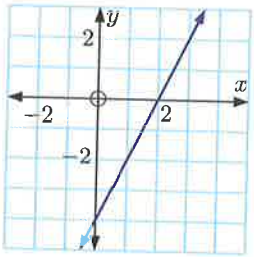
c



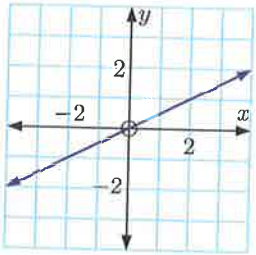
d



e

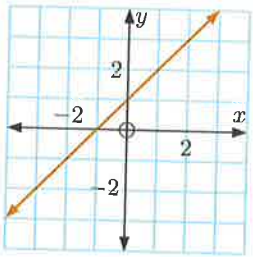


f

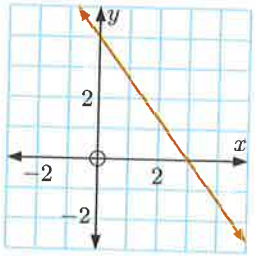


2 Find the x and y-intercepts of the following lines, and hence find the gradient of each line:

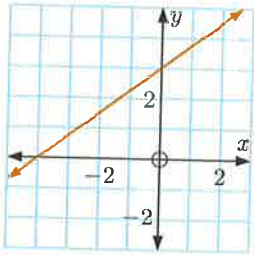
a



b



c



3 Draw the graph of the line with the following axes intercepts, and determine the line's gradient:

a x-intercept -1 and y-intercept 2

b x-intercept 2 and y-intercept 6

c x-intercept 3 and y-intercept -5

d x-intercept -4 and y-intercept -2.

DISCUSSION

Under what circumstances can we *not* determine the gradient of a line from the axes intercepts?

E THE EQUATION OF A LINE

The **equation of a line** is a rule which connects the x and y-coordinates of **all** points on the line.

For any line that is not horizontal or vertical, the rule can be written in the form $y = mx + c$ where m and c are constants.

Consider the line with equation $y = 2x$.

For all points on the line, the y-coordinate is double the x-coordinate.

Example 9 Self Tutor

State in words, the meaning of the equation $y = 3x + 7$.

$y = 3x + 7$ connects the x and y-coordinates of every point on a straight line.

For any point on the line, the y-coordinate is three times the x-coordinate, plus 7.

A **table of values** is often used to display the coordinates (x, y) for points on a line.

For example, this table of values corresponds to the line with equation $y = 2x$. It tells us that the points $(-2, -4)$, $(-1, -2)$, $(0, 0)$, $(1, 2)$, $(2, 4)$, and $(3, 6)$ all lie on the line.

x	-2	-1	0	1	2	3
y	-4	-2	0	2	4	6

Example 10 Self Tutor

By inspection only, find the equation of the straight line passing through the following points:

a

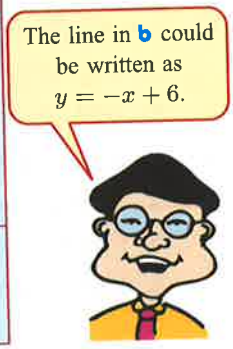
x	1	2	3	4	5
y	-3	-6	-9	-12	-15

b

x	1	2	3	4	5
y	5	4	3	2	1

a Each y-value is -3 times its corresponding x-value, so $y = -3x$.

b The sum of each x and y-value is always 6, so $x + y = 6$.



A point lies on a line if, and only if, its coordinates satisfy the equation of the line.

Example 11 Self Tutor

Determine whether $(2, 5)$ and $(-1, -5)$ lie on the line with equation $y = 4x - 3$.

The point on the line with x-coordinate 2, has y-coordinate $= 4 \times 2 - 3$
$= 8 - 3$
$= 5$ ✓

So, $(2, 5)$ does lie on the line.

The point on the line with x-coordinate -1, has y-coordinate $= 4 \times (-1) - 3$
$= -4 - 3$
$= -7$

Since the y-coordinate $\neq -5$, $(-1, -5)$ does not lie on the line.

EXERCISE 14E

1 State in words, the meaning of the equation:

a $y = x + 3$ **b** $y = 5x$ **c** $y = 2x - 6$ **d** $x + y = 5$

2 By inspection only, find the equation of the straight line passing through the following points:

a

x	1	2	3	4	5
y	4	8	12	16	20

b

x	2	3	4	5	6
y	4	5	6	7	8

c

x	0	1	2	3
y	0	-2	-4	-6

d

x	0	1	2	3
y	3	2	1	0

e

x	1	2	3	4	5
y	3	5	7	9	11

f

x	1	2	3	4	5
y	1	3	5	7	9

3 Determine whether the point:

a $(3, 4)$ lies on the line $y = 2x - 2$ **b** $(-1, 4)$ lies on the line $y = x + 6$

c $(-2, 10)$ lies on the line $y = -3x + 4$ **d** $(\frac{1}{2}, -6)$ lies on the line $y = 2x - 8$.

4 Consider the line $y = 2x - 3$.

a Find the y-coordinate of the point on the line with x-coordinate 4.

b Find the x-coordinate of the point on the line with y-coordinate -4.

5 Consider the line $y = 5 - 2x$.

a Find the y-coordinate of the point on the line with x-coordinate -2.

b Find the x-coordinate of the point on the line with y-coordinate -3.

6 Consider the line $y = 4 + x$.

a Find the y-coordinate of the point on the line with x-coordinate 0.

b Find the x-coordinate of the point on the line with y-coordinate 0.

c Hence state the axes intercepts of the line.

F GRAPHING LINES FROM EQUATIONS

If we are given the equation of a line, we can draw its graph:

- by constructing a **table of values**
- using its **gradient** and ***y*-intercept**.

GRAPHING LINES BY CONSTRUCTING A TABLE OF VALUES

For any given value of x, we can use the equation of the line to find the corresponding value of y.

Consider the equation $y = \frac{1}{2}x - 1$.

When $x = -2$, $y = \frac{1}{2}(-2) - 1$
$= -1 - 1$
$= -2$

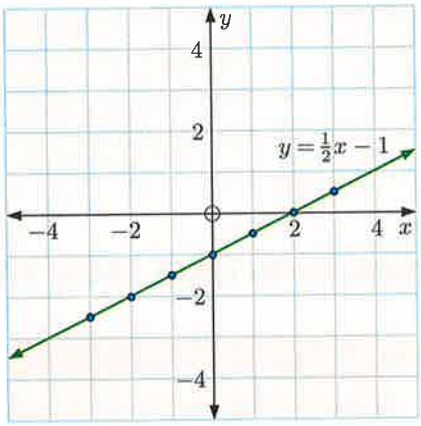
When $x = 2$, $y = \frac{1}{2}(2) - 1$
$= 1 - 1$
$= 0$

By continuing this process, we construct the **table of values**:

x	-3	-2	-1	0	1	2	3
y	$-2\frac{1}{2}$	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$

The points $(-3, -2\frac{1}{2})$, $(-2, -2)$, $(-1, -1\frac{1}{2})$, $(0, -1)$, $(1, -\frac{1}{2})$, $(2, 0)$, and $(3, \frac{1}{2})$ all satisfy $y = \frac{1}{2}x - 1$ and lie on its graph.

The graph of $y = \frac{1}{2}x - 1$ is the line which passes through these points.



Example 12 Self Tutor

Consider the equation $y = x - 2$.

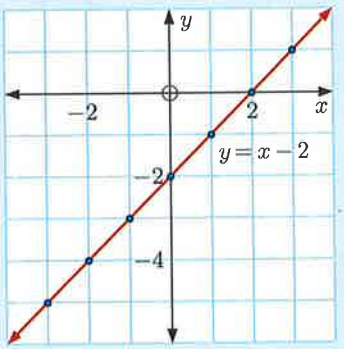
a Construct a table of values using $x = -3, -2, -1, 0, 1, 2$, and 3.

b Hence draw the graph of $y = x - 2$.

a

x	-3	-2	-1	0	1	2	3
y	-5	-4	-3	-2	-1	0	1

b



EXERCISE 14F.1

1 For each of the following equations:

i construct a table of values using $x = -3, -2, -1, 0, 1, 2, 3$

ii draw the graph of the straight line.

a	$y = x$	**b**	$y = -x$	**c**	$y = 2x$
d	$y = -2x$	**e**	$y = 2x - 1$	**f**	$y = -2x - 1$
g	$y = \frac{1}{4}x + 2$	**h**	$y = -\frac{1}{4}x + 2$	**i**	$y = 2 - x$
j	$y = 3x - 2$	**k**	$y = 1 - 2x$	**l**	$y = -1.5x - 1$


GRAPHICS CALCULATOR INSTRUCTIONS

Use your **graphics calculator** or the **graphing package** to check your answers.


GRAPHING PACKAGE

2 For each of the following equations:

i construct a table of values using $x = -3, -2, -1, 0, 1, 2, 3$

ii draw the graph of the straight line

iii state the gradient and y-intercept of the line.

a $y = x + 3$ **b** $y = -2x + 1$ **c** $y = \frac{1}{3}x - 1$

GRAPHING LINES FROM THE GRADIENT AND y-INTERCEPT

INVESTIGATION GRAPHING LINES

In this Investigation, we discover how the equation of a line relates to the *gradient* and *y-intercept* of the line.

What to do:

GRAPHING PACKAGE



1 Draw the graph of each line given. You can do this using a **table of values**, your **graphics calculator**, or the **graphing package**. Hence write down the gradient and y-intercept of each line.

To find the gradient of the line, choose any two points on the line and use the gradient formula.



	Equation of line	*Gradient*	*y-intercept*
a	$y = 2x + 1$	2	1
b	$y = 3x - 2$		
c	$y = -x + 3$		
d	$y = 4x$		
e	$y = -2x + 2$		
f	$y = \frac{1}{2}x - 1$		
g	$y = -\frac{1}{3}x + 1$		

2 Copy and complete: "For a line with equation $y = mx + c$, the gradient is ..... and the y-intercept is ..... ."

You should have discovered that:

> $y = mx + c$ is the equation of a straight line with gradient m and y-intercept c.

Example 13 Self Tutor

State the gradient and y-intercept of the line with equation:

a $y = 3x - 2$ **b** $y = 7 - 2x$ **c** $y = 0$

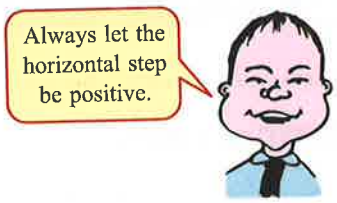
a $y = 3x - 2$ has $m = 3$ and $c = -2$
$\therefore$ the gradient is 3 and the y-intercept is -2.

b $y = 7 - 2x$ can be written as $y = -2x + 7$, with $m = -2$ and $c = 7$
$\therefore$ the gradient is -2 and the y-intercept is 7.

c $y = 0$ can be written as $y = 0x + 0$, with $m = 0$ and $c = 0$
$\therefore$ the gradient is 0 and the y-intercept is 0.

To draw the graph of $y = mx + c$:

- Use the y-intercept c to plot the point $(0, c)$.
- Starting from $(0, c)$, use horizontal and vertical steps from the gradient m to locate another point on the line.
- Join the two points and extend the line in either direction.

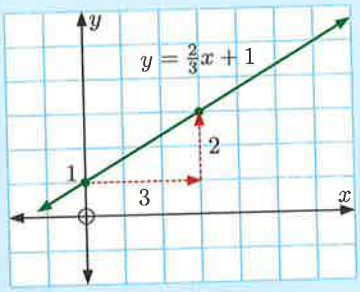


Example 14 Self Tutor

Draw the graph of: **a** $y = \frac{2}{3}x + 1$ **b** $y = -2x - 3$.

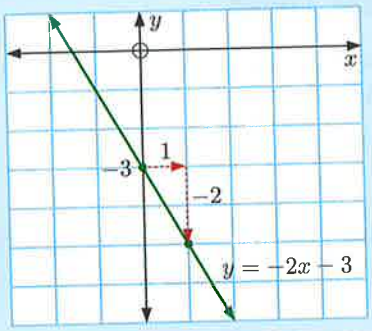
a For $y = \frac{2}{3}x + 1$:

- the y-intercept is $c = 1$
- the gradient is $m = \frac{2}{3}$ ← vertical step, ← horizontal step



b For $y = -2x - 3$:

- the y-intercept is $c = -3$
- the gradient is $m = \frac{-2}{1}$ ← vertical step, ← horizontal step



EXERCISE 14F.2

1 State the gradient and y-intercept of the line with equation:

a $y = 4x + 8$ **b** $y = -3x + 2$ **c** $y = 6 - x$

d $y = -2x + 3$ **e** $y = -2$ **f** $y = 11 - 3x$

g $y = \frac{1}{2}x - 5$ **h** $y = 3 - \frac{3}{2}x$ **i** $y = \frac{2}{5}x + \frac{4}{5}$

j $y = \frac{x+1}{2}$ **k** $y = \frac{2x - 10}{5}$ **l** $y = \frac{11 - 3x}{2}$

Equations in the form $y = mx + c$ are said to be in *gradient-intercept* form.



2 Draw the graph of:

a $y = x + 3$ **b** $y = -x + 4$ **c** $y = 2x + 2$

d $y = -3x - 2$ **e** $y = \frac{1}{2}x - 1$ **f** $y = \frac{2}{3}x + 4$

g $y = 3x$ **h** $y = -\frac{1}{2}x$ **i** $y = -2x + 1$

j $y = 3 - \frac{1}{3}x$ **k** $y = \frac{3}{4}x - 2$ **l** $y = -\frac{1}{4}x - 3$

3 Consider the line with equation $y = -\frac{2}{3}x + 2$.

a Find the: **i** gradient **ii** y-intercept **iii** x-intercept.

b Draw the graph of the line. **c** Does the point $(4, -\frac{2}{3})$ lie on the line?

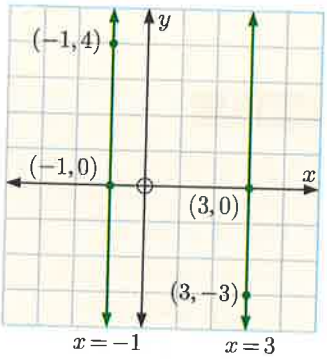
G VERTICAL AND HORIZONTAL LINES

Vertical and horizontal lines are special cases which we need to treat with care.

VERTICAL LINES

For all points on a vertical line, the x-coordinate is constant regardless of the value of the y-coordinate.

The graph alongside shows the vertical lines $x = -1$ and $x = 3$.

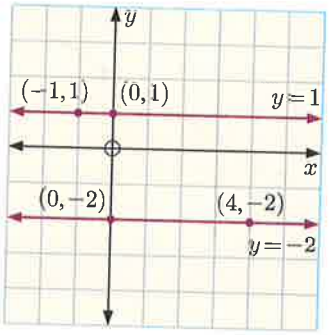


All **vertical** lines have equations of the form $x = a$.
The gradient of a vertical line is **undefined**.

HORIZONTAL LINES

For all points on a horizontal line, the y-coordinate is constant regardless of the value of the x-coordinate.

The graph alongside shows the horizontal lines $y = -2$ and $y = 1$.



All **horizontal** lines have equations of the form $y = c$.
The gradient of a horizontal line is **zero**.

EXERCISE 14G

1 Draw the graph of the line with equation:

a $x = 2$ **b** $y = 3$ **c** $y = -4$ **d** $x = -3$

e $y = 1.5$ **f** $x = -\frac{5}{2}$ **g** $y = 0$ **h** $y = -\frac{1}{4}$

2 Find the gradient of each line in **1**.

3 Suppose l_1 is the line with equation $x = 5$, and l_2 is the line with equation $y = -3$.

a Graph l_1 and l_2 on the same set of axes.

b Suppose l_1 cuts the x-axis at A, and l_2 cuts the y-axis at B. Find:

i the coordinates of A and B **ii** the gradient of [AB].

4 **a** On the same set of axes, graph the lines $x = -2$, $x = 6$, $y = 5$, and $y = -6$.

b Consider the quadrilateral formed by the intersection points of these lines.

i What type of quadrilateral is it?

ii Find the area of the quadrilateral.

iii Find the gradient of the diagonals of the quadriateral.

H FINDING EQUATIONS FROM GRAPHS

If we know the y-intercept c and one other point on a graph, then we can find the gradient m using the gradient formula.

We can then write down the equation of the line as $y = mx + c$.

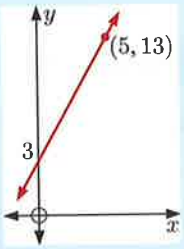
Example 15 Self Tutor

For the graph alongside, determine:

a the gradient of the line

b the y-intercept

c the equation of the line.



a $(0, 3)$ and $(5, 13)$ lie on the line

$\therefore$ the gradient $m = \dfrac{13-3}{5-0}$

$= \frac{10}{5}$

$= 2$

b The y-intercept is 3.

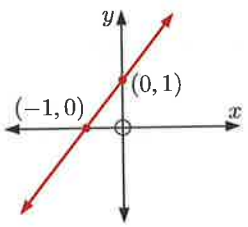
c The equation of the line is $y = 2x + 3$.

EXERCISE 14H

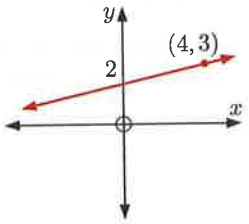
1 For each of the following lines, find:

i the gradient **ii** the y-intercept **iii** the equation of the line.

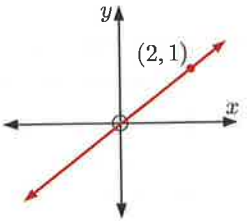
a



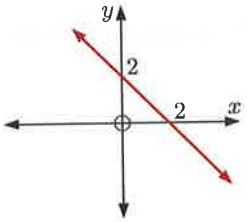
b



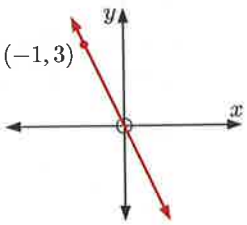
c



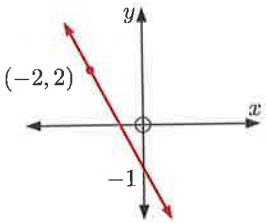
d



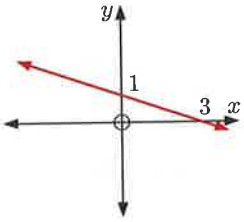
e



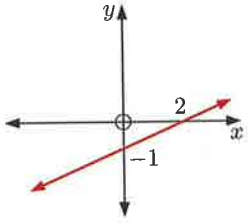
f



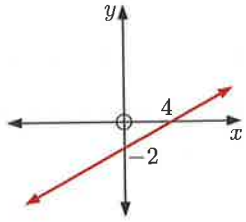
g



h

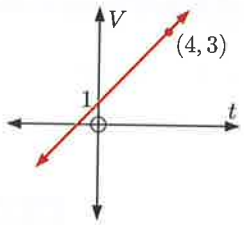


i

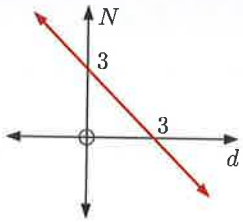


2 Find the rule connecting the variables in:

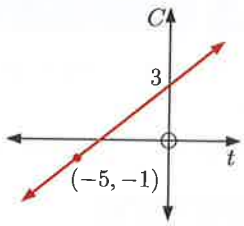
a



b

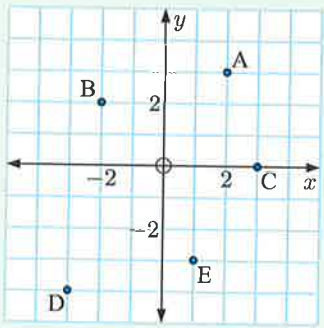


c



REVIEW SET 14A

1 State the coordinates of the points A, B, C, D, and E.



2 On a set of axes, illustrate all the points which have equal x and y-coordinates.

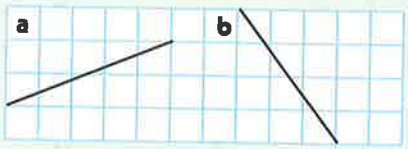
3 For the equation $y = 4 - 3x$:

- **a** construct a table of values with $x = -3, -2, -1, 0, 1, 2, 3$
- **b** draw the graph of the straight line
- **c** state the gradient and y-intercept of the line.

4 An accountant charges a £70 consultation fee and then £120 per hour thereafter.

- **a** Identify the independent and dependent variables.
- **b** Make a table of values for the cost £C of an appointment with the accountant for t hours where $t = 0, 1, 2, 3, 4$.
- **c** Draw a graph of C against t.
- **d** Is the relationship between C and t linear?
- **e** Is it sensible to join the points graphed with a straight line? Explain your answer.
- **f** For every increase in t of 1 hour, what is the change in C?

5 State the gradient of each line:



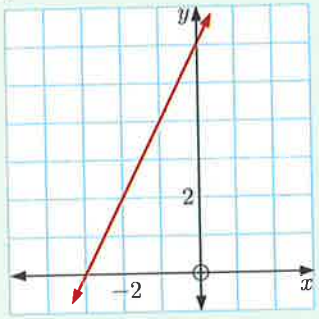
6 On the same set of axes, draw graphs of the lines with equations $x = -2\frac{1}{2}$ and $y = -4$.

7 State the gradient and y-intercept of the line with equation:

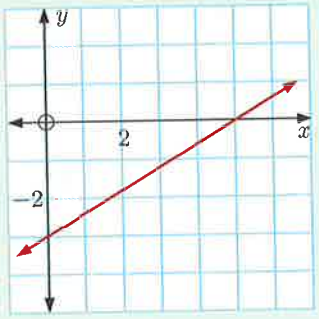
- **a** $y = 4x - 3$
- **b** $y = 2 - x$
- **c** $y = \dfrac{3x - 1}{2}$

8 Find the x and y-intercepts of each of the following lines, and hence find the gradient of the line:

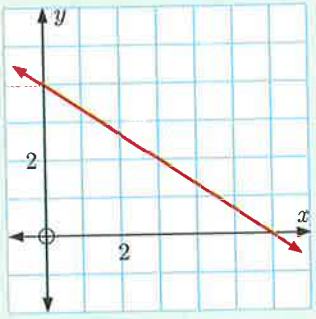
a



b



c



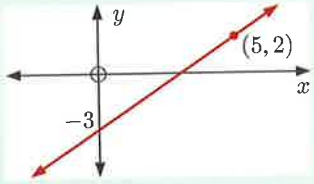
9 For each of the following lines:

i state the gradient and y-intercept **ii** draw the graph of the line.

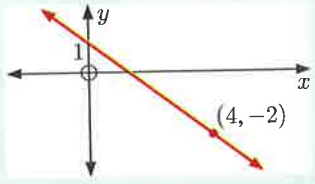
a $y = 2x - 3$ **b** $y = \frac{1}{3}x + 1$ **c** $y = 8 - 3x$

10 Find the equation of each line:

a



b



REVIEW SET 14B

1 Plot the points P(2, 5), Q(3, −3), R(−4, 0), and S(−2, −4) on a set of axes.

2 Find, by inspection, the equation of the straight line passing through these points:

x	0	1	2	3	4	5
y	3	2	1	0	−1	−2

3 A tank contains 400 litres of water. The tap is left on and 20 litres of water escape per minute. Suppose V is the volume of water remaining in the tank t minutes after the tap is turned on.



a What are the independent and dependent variables?

b Make a table of values for V and t, for $t = 0$, 2, 4, 6, 8, 10 minutes.

c Draw the graph of V against t.

d Is the relationship between V and t linear?

e Is it sensible to join the points graphed with a straight line? Explain your answer.

f For every increase of 1 minute for t, what is the change in V?

g How much water is left in the tank after $2\frac{1}{2}$ minutes?

h How long will it take for the volume of water in the tank to fall to 310 litres?

4 On grid paper, draw a line with a gradient of:

a $\frac{2}{5}$ **b** $-\frac{4}{3}$ **c** 0

5 Find the gradient of the straight line through the points:

a $(2, 1)$ and $(8, 13)$ **b** $(-2, 3)$ and $(1, -4)$

c $(-1, -5)$ and $(9, 3)$ **d** $(2, -3)$ and $(-7, 12)$.

6 Draw a line through the point:

a $(3, 5)$ with gradient $\frac{1}{4}$ **b** $(-2, 3)$ with gradient -2.

7 Determine whether the point $(-2, 5)$ lies on the line with equation $y = -2x + 1$.

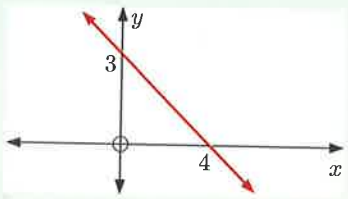
8 For each of the following lines:

i state the gradient and y-intercept **ii** draw the graph of the line.

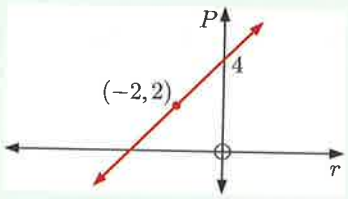
a $y = -\frac{1}{3}x + 2$ **b** $y = \frac{3}{4}x - 3$

9 Find the equation of each line:

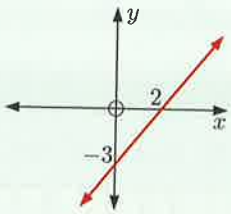
a



b



10



For the line given, find the:

a x-intercept

b y-intercept

c equation of the line.