afgeleide rekenregels

www.karelappeltans.be

July 29, 2020

1 Afgeleide machtsfunctie

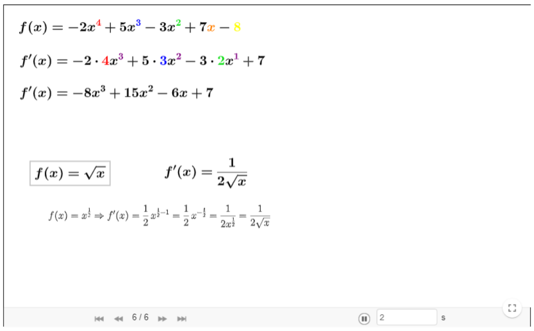


Figure 1: https://www.geogebra.org/m/rkbXbnRv

$$\boxed{(x^n)' = nx^{n-1}}$$

2 Afgeleide rationale functies

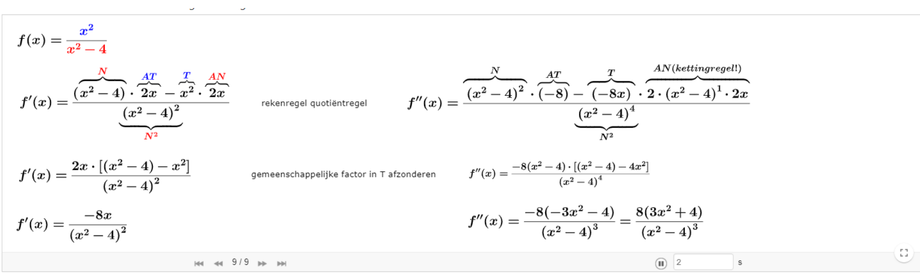


Figure 2: https://www.geogebra.org/m/MpFEGPfT

$$\boxed{\left(\frac{T}{N}\right)' = \frac{NAT - TAN}{N^2}}$$

3 Afgeleide irrationale functies

$$\boxed{(\sqrt{x})' = \frac{1}{2\sqrt{x}}}$$

4 Afgeleide goniometrische functies

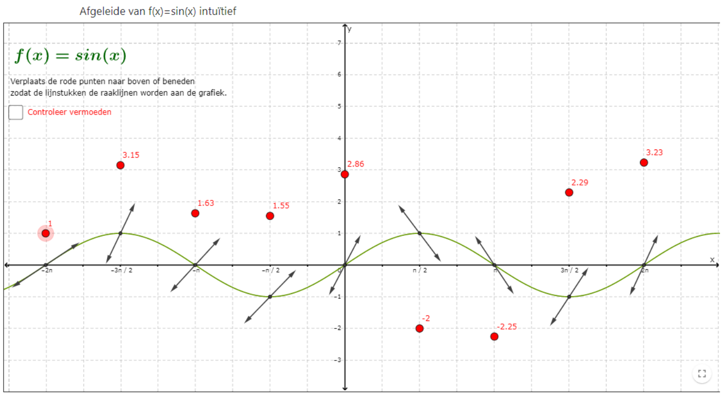


Figure 3: https://www.geogebra.org/m/qkq5rdwr

$$\boxed{(\sin(x))'=\cos(x)}$$

$$\boxed{(\cos(x))'=-\sin(x)}$$

$$\boxed{(\tan(x))'=1\frac{}{\cos^2(x)}}$$

5 Algemene rekenregels

5.1 De somregel

$$\boxed{(a \cdot f(x) + b \cdot g(x))' = a \cdot f'(x) + b \cdot g'(x)}$$

$$(3sin(x) + 5\sqrt{x})' = 3cos(x) + \frac{5}{2\sqrt{x}}$$

5.2 De productregel

$$\boxed{(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)}$$

$$\left(x^2 \cdot cos(x)\right)' = 2x \cdot cos(x) + x^2 \cdot (-sin(x))$$

5.3 De quotiëntregel

$$\boxed{\left(\frac{1}{f(x)}\right)' = \frac{-f'(x)}{f(x)^2}}$$

$$\left(\frac{1}{sin(x)}\right)' = \frac{-cos(x)}{sin^2(x)}$$

$$\boxed{\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}}$$

$$\left(\frac{2x^3 + 4x^2 + 1}{sin(x)}\right)' = \frac{sin(x) \cdot (6x^2 + 8x) - (2x^3 + 4x^2 + 1) \cdot cos(x)}{sin^2(x)}$$

5.4 De kettingregel

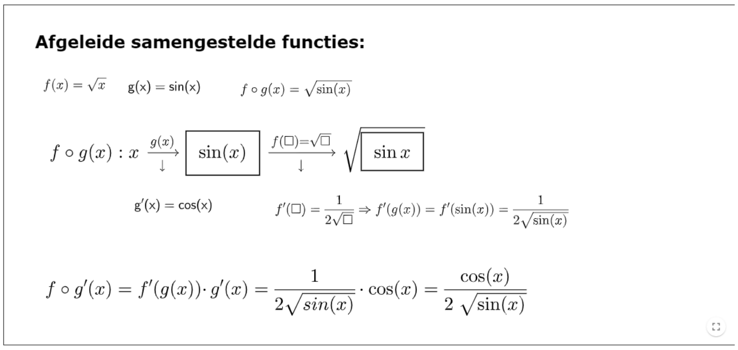


Figure 4: https://www.geogebra.org/m/YN8wHwSh

$$\boxed{(f(g(x))' = f'(g(x)) \cdot g'(x)}$$

$$\left(sin(x^2+3)\right)' = cos(x^2+3) \cdot 2x$$

5.5 Afgeleide inverse functie

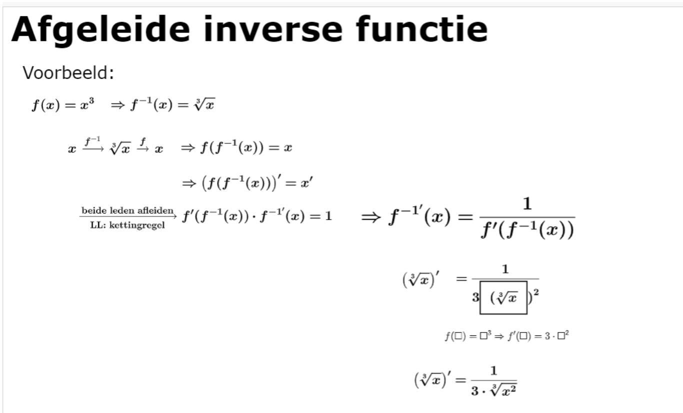


Figure 5: https://www.geogebra.org/m/hwzhaqsb

$$\boxed{\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}}$$

$$\left(\sqrt[3]{x}\right)' = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3(\sqrt[3]{x^2})}$$

6 Overzicht rekenregels

$(af+bg)'(x) = af'(x) + bg'(x) \quad (a, b \in \mathbb{R})$ $\qquad (\square + \triangle)' = \square' + \triangle'$

$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ $\qquad (\square \cdot \triangle)' = \square' \cdot \triangle + \square \cdot \triangle'$

$\left(f^r\right)'(x) = r f^{r-1}(x) \cdot f'(x) \quad (r \in \mathbb{R})$ $\qquad \left(\square^r\right)' = r\,\square^{r-1} \cdot \square'$

$\left(\frac{1}{f}\right)' = \frac{-f'(x)}{f(x)^2}$ $\qquad \left(\frac{1}{\square}\right)' = \frac{-\square'}{\square^2}$

$\left(\frac{f}{g}\right)'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$ $\qquad \left(\frac{\square}{\triangle}\right)' = \frac{\square' \cdot \triangle - \square \cdot \triangle'}{\triangle^2}$

$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$

$\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))}$

$\vdots$

⋮

$f(x) = c \ (c \in \mathbb{R})$	$f'(x) = 0$	
$f(x) = x$	$f'(x) = 1$	
$f(x) = x^r \ (r \in \mathbb{R})$	$f'(x) = rx^{r-1}$	$(\square^r)' = r \cdot \square^{r-1} \cdot \square'$
$f(x) = \sqrt{x}$	$f'(x) = \frac{1}{2\sqrt{x}}$	$\left(\sqrt{\square}\right)' = \frac{1}{2\sqrt{\square}} \cdot \square'$
$f(x) = e^x$	$f'(x) = e^x$	$(e^\square)' = e^\square \cdot \square'$
$f(x) = a^x \ (a \in \mathbb{R}_0^+ \setminus \{1\})$	$f'(x) = a^x lna$	$(a^\square)' = a^\square lna \cdot \square'$
$f(x) = lnx$	f'(x)=$\frac{1}{x}$	$(ln\square)' = \frac{1}{\square} \cdot \square'$
$f(x) = \log_a x$	$f'(x) = \frac{1}{x \ln a}$	$(\log_a \square)' = \frac{1}{\square \ln a} \cdot \square'$
$f(x) = \sin x$	$f'(x) = \cos x$	$(sin\square)' = \cos \square \cdot \square'$
$f(x) = \cos x$	$f'(x) = -\sin x$	$(\cos \square)' = -\sin \square \cdot \square'$
$f(x) = \tan x$	$f'(x) = \frac{1}{\cos^2 x}$	$(\tan \square)' = \frac{1}{cos^2\square} \cdot \square'$
$f(x) = \cot x$	$f'(x) = \frac{-1}{\sin^2 x}$	$(\cot \square)' = \frac{-1}{sin^2\square} \cdot \square'$
$f(x) = \arcsin x$	$f'(x) = \frac{1}{\sqrt{1-x^2}}$	$(\arcsin \square)' = \frac{1}{\sqrt{1-\square^2}} \cdot \square'$
$f(x) = \arccos x$	$f'(x) = \frac{-1}{\sqrt{1-x^2}}$	$(\arccos \square)' = \frac{-1}{\sqrt{1-\square^2}} \cdot \square'$
$f(x) = \arctan x$	$f'(x) = \frac{1}{1+x^2}$	$(\arctan \square)' = \frac{1}{1+\square^2} \cdot \square'$
$f(x) = \mathrm{arc}\cot x$	$f'(x) = \frac{-1}{1+x^2}$	$(\mathrm{arc}\cot \square)' = \frac{-1}{1+\square^2} \cdot \square'$

7 Oefeningen

1. Op deze site kunnen alle oefeningen gecontroleerd worden: https://www.derivative-calculator.net/

2. afgeleide samengestelde functies: https://www.geogebra.org/m/YN8wHwSh

3. Bereken de afgeleide van onderstaande functies

   (a) $f(t) = (1+t)\ln t$
   (b) $f(t) = t^2 \cos t$
   (c) $f(t) = \dfrac{-1}{t^2}$
   (d) $f(t) = \dfrac{3t-1}{2t+2}$
   (e) $f(x) = \sin(4x+5)$
   (f) $f(t) = \ln(7t^2)$
   (g) $f(x) = \dfrac{4x^3+1}{3x}$
   (h) $f(x) = \tan \dfrac{1}{x}$
   (i) $f(x) = \cos(-8x^2-1)$
   (j) $f(x) = \sin^3(3x)$
   (k) $f(x) = \sqrt{x^2-7x+8}$
   (l) $f(t) = \ln t \, \sin(t^2)$
   (m) $f(t) = \ln(1-t^2)$

4. Bereken de afgeleide van onderstaande functies

   (a) $r(\theta) = 2\,\theta^{1/2} + \dfrac{3}{2}\theta^{2/3} + \dfrac{4}{3}\theta^{3/4}$
   (b) $r(\theta) = \dfrac{4}{1+2\cos\theta}$
   (c) $r(\theta) = \sqrt{1-2\theta}$
   (d) $r(\theta) = \dfrac{a}{\theta}$ met $a > 0$.

5. alle rekenregels door elkaar:

   (a) https://homepages.bluffton.edu/~nesterd/apps/derivs.html
   (b) https://www.math-exercises.com/limits-derivatives-integrals/derivative-of-a-function

6. Bepaal de waarden van $n \in \mathbb{Z}$ zodat $y = x^n$ een oplossing is van volgende (differentiaal)vergelijking: $x^2y" - 2xy' = 4y$ (A: n=4 en n=-1)

7. Bereken de gevraagde afgeleide m.b.v. onderstaande tabel

Use the following table to answer question questions # 9 and # 10.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	1	3	2
2	2	2	1	3
3	3	1	3	1

9. If $h(x) = f\big(g(x)\big)$, what is $h'(1)$?

10. If $H(x) = g\big(f(x)\big)$, what is $H'(3)$?

8. Bereken de gevraagde afgeleide m.b.v. onderstaande tabel

t	0	1	2	3	4
$h(t)$	-2	2	3	4	8
$h'(t)$	3.5	0.5	2.5	1.5	5
$h''(t)$	6	0.25	0.3	-0.4	0.6

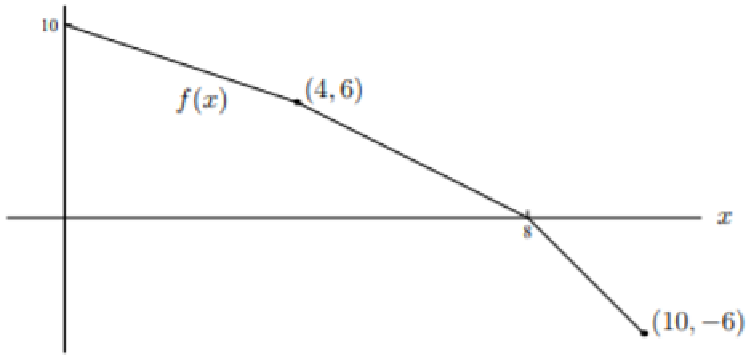
(a) $a(t) = h(t^2 - 1)$ gevraagd $a'(3)$

(b) $b(t) = \frac{h(t)}{t^2}$ gevraagd $b'(4)$

(c) $c(t) = h^{-1}(t)$ gevraagd $c'(2)$

9. Bepaal g'(3) als

Consider the piecewise linear function $f(x)$ graphed below:



(a) $g(x) = \frac{f(x^2)}{x}$ (antw. $-\frac{51}{9}$)

(b) $g(x) = \sin\big(f(x)^3\big)$ (antw. $-147cos(343)$)

(c) $g(x) = f^{-1}(x)$ (antw. $\frac{-2}{3}$)

8 Taken

1. taak rekenregels

2. taak kettingregel