Nice pair − wise quadratic polynomials.

Two quadratic real polynomials is called a nice pair of quadratic polynomials if they fulfill the conditions below.

A: They have two intersection points.

B: Both polynomials have two roots and all four roots are distinct.

C: All the values associated with the three points below are integers:

1: *Roots*, 2: *Intersection points*, 3: *Coefficients and the constant c.*

A nice pair of quadratic polynomials is called a NPWP2 polynomial.

Two parametrized subsets of NPWP2 polynomials.

Theorem.

Let $(a, b, r) \in \mathbb{N}^3, a > b, r > 1, (q, k, s) \in \mathbb{Z}^3, s \neq 0.$

A: Let $f(x)$ *and* $g(x)$ *be given by*:

$f(x) = sr^2(k + x)(-4\,a\,b\,q\,(r^2 - 1) + k + x)$

$g(x) = s(r^2 - 1)(-2abq\,(r^2 - 1) + k + x)(-2\,abq(r^2 - 1) - 2qr(a^2 - b^2) + k + x)$

B: Let $f(x)$ *and* $g(x)$ *be given by*:

$f(x) = r^2 s\,(k + x)(-2\,q\,(a^2 - b^2)(r^2 - 1) + k + x)$

$g(x) = s\,(r^2 - 1)(-q(a^2 - b^2)(r^2 - 1) + k + x)(-q\,(a^2 - b^2)(r^2 - 1) - 4abqr + k + x)$

These parametrizations are subsets of the set of NPWP2 polynomials.

Proof.

The proof of A is made with simple substitutions and the wellknown parametrization formula for Pythagorean triples. See the pictures in endnotes: [i] [ii]

The proof of B is similar to the proof of A.

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Some examples.

$f(x) = 4x \cdot (-24 + x)$	$g(x) = 3 \cdot (-12 + x)(0 + x)$
$f(x) = 9x \cdot (-64 + x)$	$g(x) = 8 \cdot (-32 + x)(-14 + x)$
$f(x) = 16x \cdot (-120 + x)$	$g(x) = 15 \cdot (-60 + x)(-36 + x)$
$f(x) = 25x \cdot (-192 + x)$	$g(x) = 24 \cdot (-96 + x)(-66 + x)$
$f(x) = 36x \cdot (-280 + x)$	$g(x) = 35 \cdot (-140 + x)(-104 + x)$
$f(x) = 49x \cdot (-384 + x)$	$g(x) = 48 \cdot (-192 + x)(-150 + x)$
$f(x) = 64x \cdot (-504 + x)$	$g(x) = 63 \cdot (-252 + x)(-204 + x)$
$f(x) = 81x \cdot (-640 + x)$	$g(x) = 80 \cdot (-320 + x)(-266 + x)$
$f(x) = 100x \cdot (-792 + x)$	$g(x) = 99 \cdot (-396 + x)(-336 + x)$

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