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(23) Calcule a integral de linha ao longo da curva C:

$$\int_C (x + 2y)\,dx + (x - y)\,dy$$

C: $x = 2\cos t$, $y = 4\operatorname{sen} t$ $\quad (0 \le t \le \pi/4)$

$$\int_0^{\pi/4} [(2\cos t + 8\operatorname{sen} t).(-2\operatorname{sen} t) + (2\cos t - 4\operatorname{sen} t)(4\cos t)]\,dt$$

$$\int_0^{\pi/4} [-4\cos t \operatorname{sen} t - 16\operatorname{sen}^2 t + 8\cos^2 t - 16\operatorname{sen} t \cos t]\,dt$$

$$\int_0^{\pi/4} -20\operatorname{sen} t \cos t - 16\operatorname{sen}^2 t + 8\cos^2 t \;\; dt$$

Usando $2\operatorname{sen}(t)\cos(t) = \operatorname{sen}(2t)$

$$\Rightarrow \int -10\operatorname{sen}(2t) - 16\operatorname{sen}^2 t + 8\cos^2 t \;\; dt$$

$$\Rightarrow -\int 10\operatorname{sen}(2t)\,dt - \int 16\operatorname{sen}^2 t\,dt + \int 8\cos^2 t\,dt$$

$$\Rightarrow 5\cos(2t) - 8t + 4\operatorname{sen}(2t) + 4t + 2\operatorname{sen}(2t)$$

$$\Rightarrow 5\cos(2t) - 4t + 6\operatorname{sen}(2t)$$

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$\Rightarrow 5\cos(2t) - 4t + 6\,\text{sen}(2t) \Big|_{0}^{\pi/4}$

$\Rightarrow 5\cos\left(2\cdot\frac{\pi}{4}\right) - 4\left(\frac{\pi}{4}\right) + 6\,\text{sen}\left(2\cdot\frac{\pi}{4}\right) - \left[5\cos(0) + 6\,\text{sen}(0)\right]$

$\Rightarrow -\pi + 1$ //