Coterminal Angles

Coterminal angles: are angles in standard position (angles with the initial side on the positive *x*-axis) that have a common terminal side. For example, the angles 30°, –330° and 390° are all coterminal (see figure 2.1 below).

Fig. 2.1

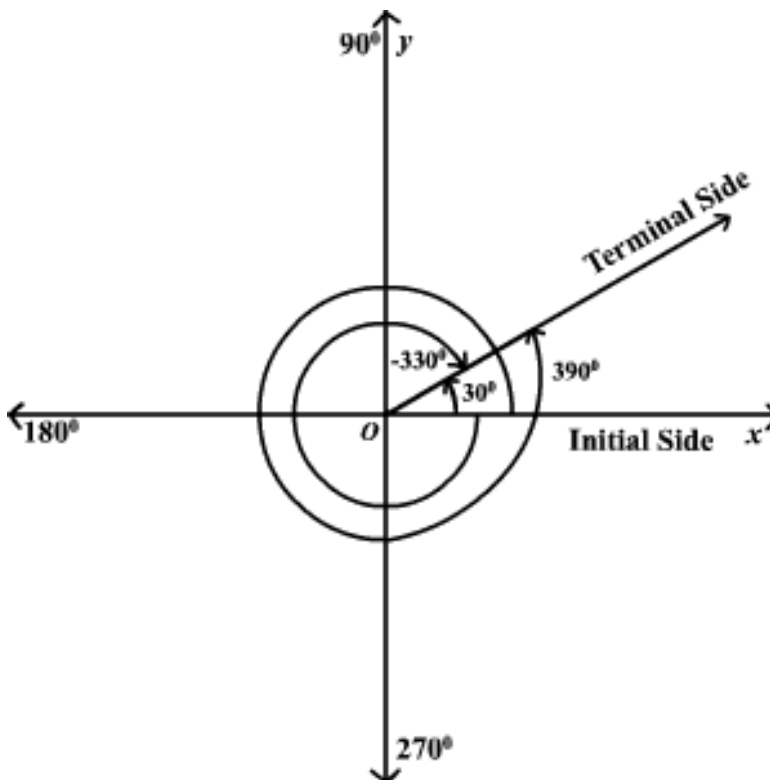In general, if θ is any angle, then **θ + n(360)** is coterminal angle with θ**,** for all nonzero integer **n**. For **positive** angle θ**, the coterminal angle** can be found by: θ **+** 360°

Example 2.1: Find three positive angles that are coterminal with

30°

-55°

Solution: Use the formula **θ + n(360)** as follows:

30° + 360° = **390°**
30° + 2(360°)= **750°**
30° + 3(360°)= **1110°**

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Note: There are an infinite number of angles coterminal with 30°.

-55° + 360°= **305°**
-55° + 2(360°)= **665°**
-55° + 3(360°)= **1025°**

Example 2.2

Determine the measures of angles that are coterminal with the given angle in the stated domain.

a) $\theta = 75°$ for $-500° \leq \theta \leq 500°$

b) $\theta = -105°$ for $-600° \leq \theta \leq 600°$

Solution:

a) For θ between 500° and 0°, the coterminal angles are 75° and 75° + 360°= 435°

   For θ between 0° and - 500°, the coterminal angle is 75° - 360° = -285°

b) For θ between 600° and 0°, the coterminal angle is -105° + 360° = 255°

   For θ between 0° and -600°, the coterminal angles are: -105°; and -105°- 360° = -465°

Practice Exercises:

1. Find an angle between 0° and 360° ($0° \leq \theta \leq 360°$) that is coterminal with the given angles.

   a) 1190°   b)-740°   c) 743°   d) -500°

2. Find an angle between -360° and 0° ($-360° \leq \theta \leq 0°$) that is coterminal with the given angles.

   a) 1190°   b) -740°   c) 743°   d) -500°

Answers:

1. a) 110°   b) 340°   c) 23°   d) 220°
2. a) -250°   b) -20°   c) -337°   d) -140°