Visualization of each Step and the Solution of Gauss-Jordan Elimination using GeoGebra

Connecting the 3D visualization of each algebraic step and the solution of Gauss-Jordan Elimination through an interactive GeoGbra based applet.

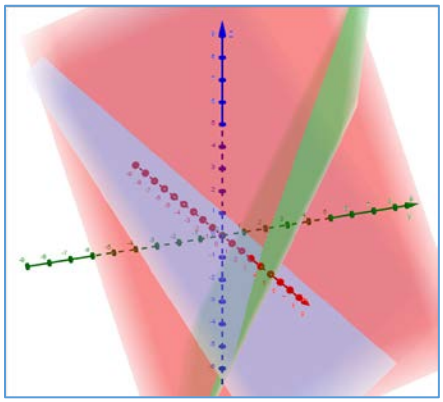
Author: James D. Factor

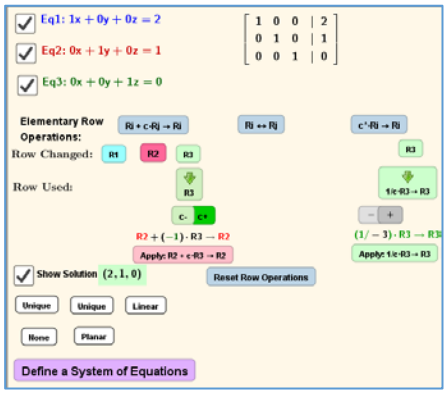
This presentation will use an interactive GeoGebra applet to show the visual geometric changes for each step of the Gaussian–Jordan elimination process as the solution unfolds in achieving reduced row echelon form. The cases of one solution, no solution, and infinite solutions (linear and planar) will be illustrated dynamically.

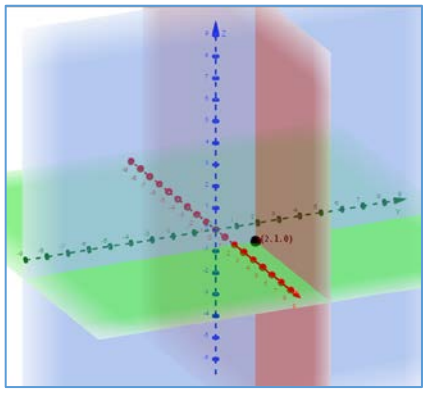
This applet, along with others, is freely available for instructor demonstration and student discovery of the meaning of linear algebra concepts. Associated activities have been designed to guide students in using the interactive applets to enhance learning. This work is part of the NSF project entitled *Transforming Linear Algebra Education with GeoGebra Applets* (NSF TUES Grant DUE-1141045). Additional information about the project developed applets is available at the MAA/NSF Poster Session.

Gauss-Jordan Elimination Examples

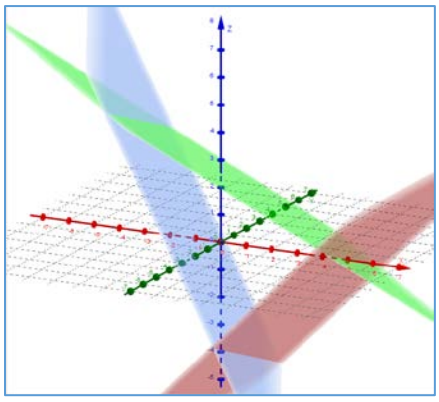
One solution

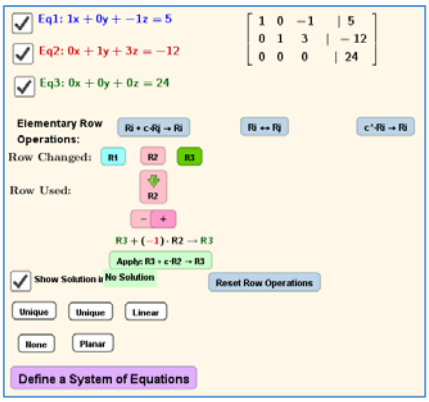


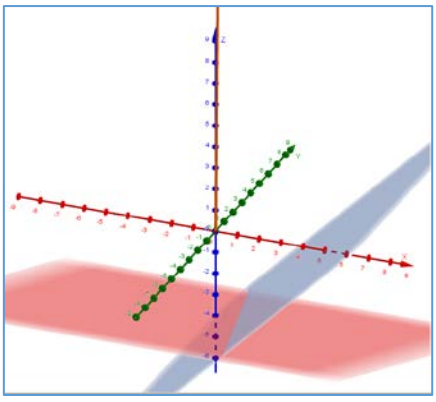




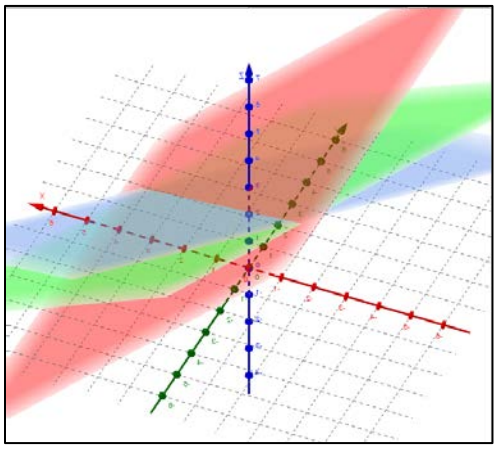
No Solution

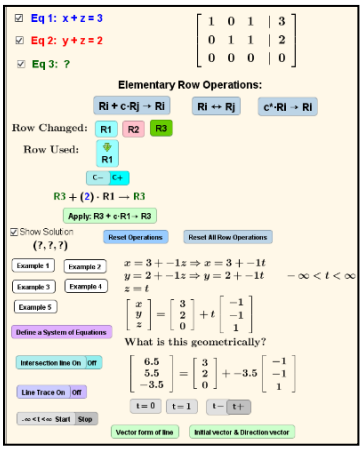


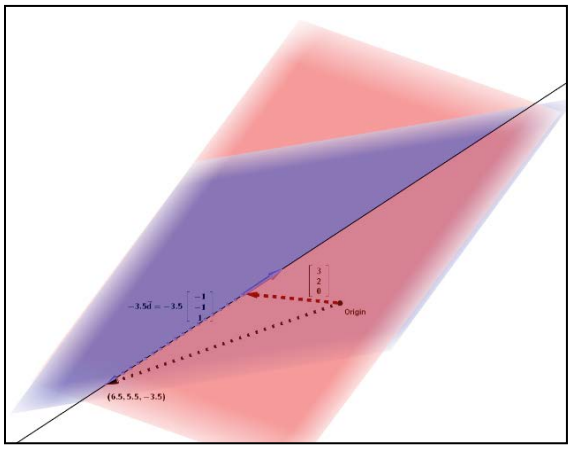




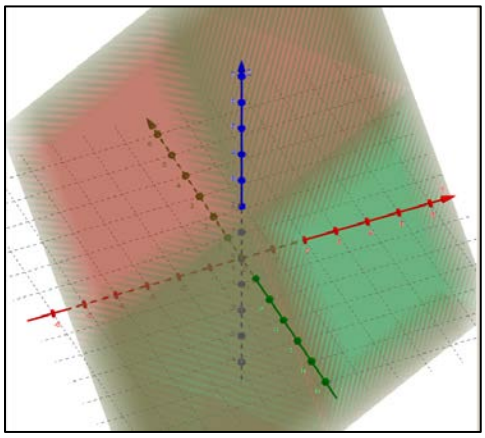
Infinitely many solutions (Linear)

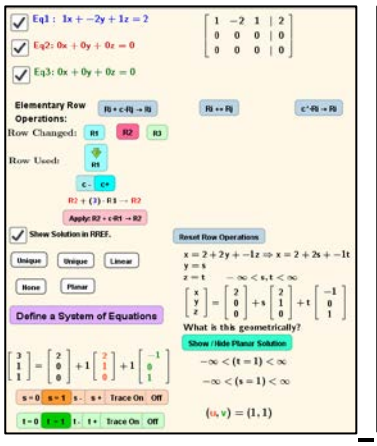


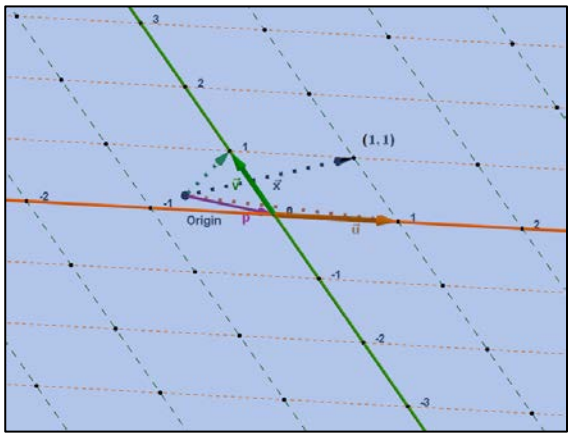




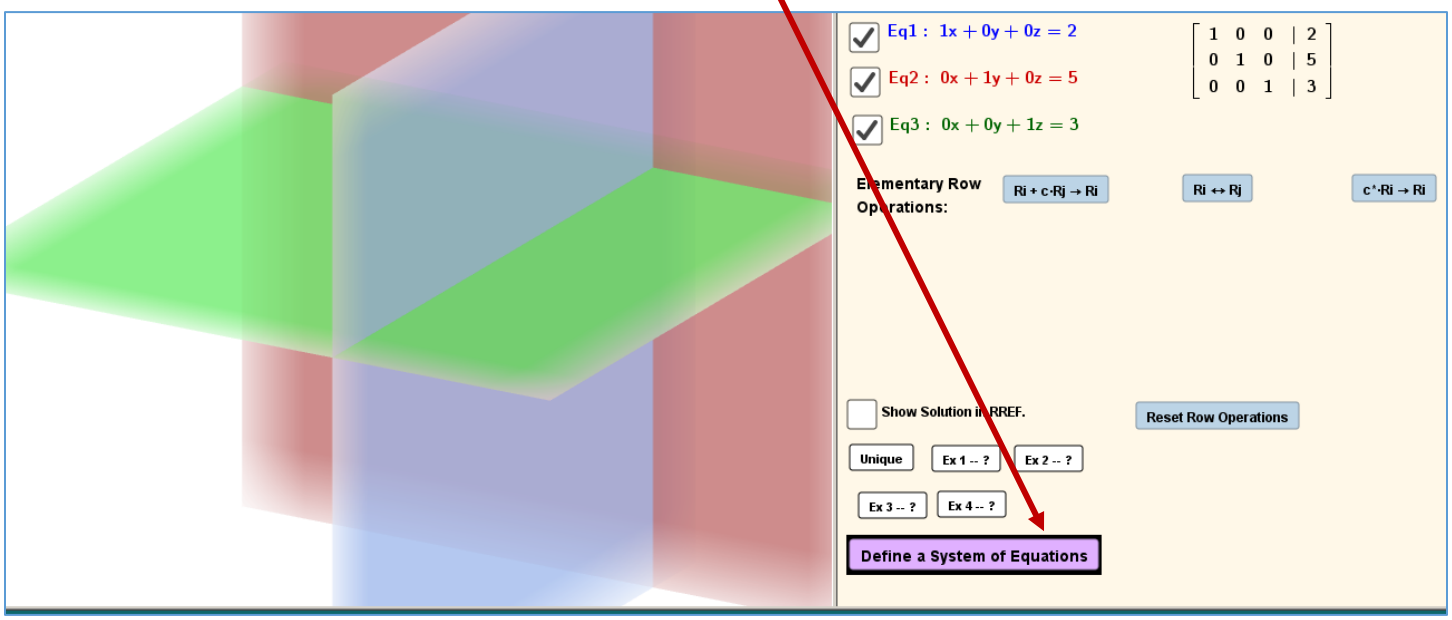
Infinitely many solutions (Planar)



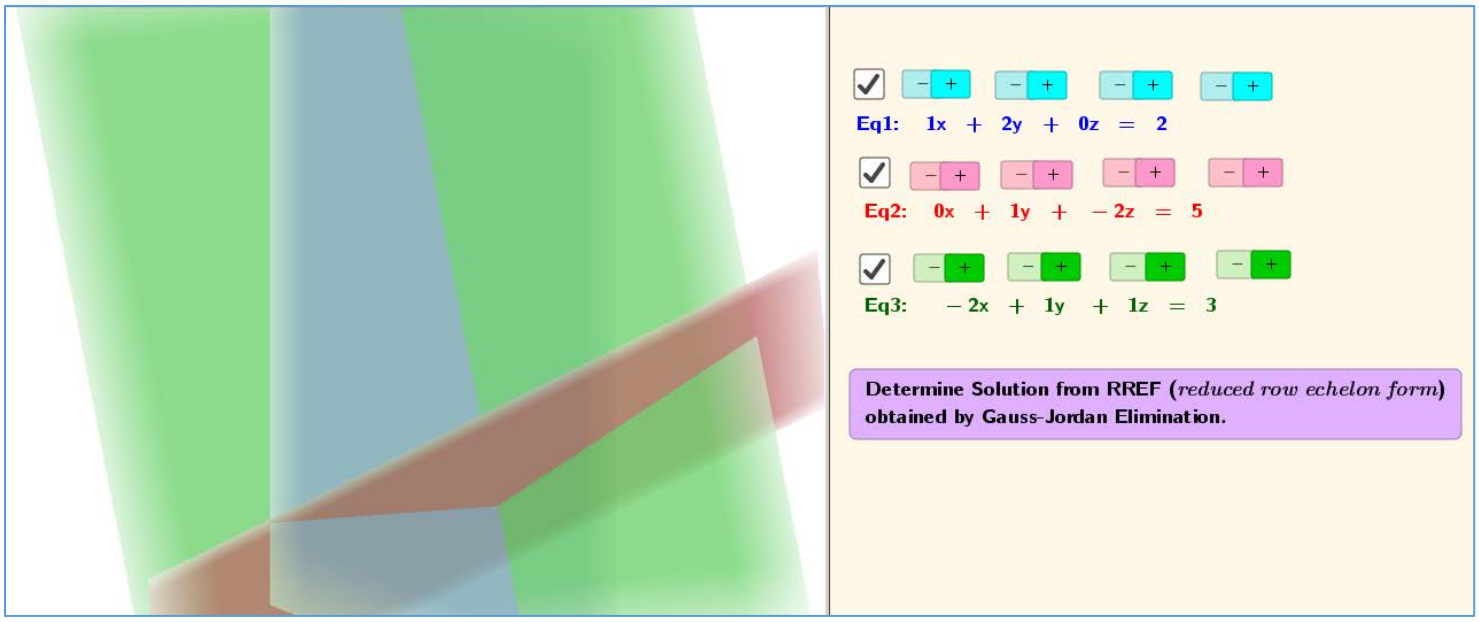


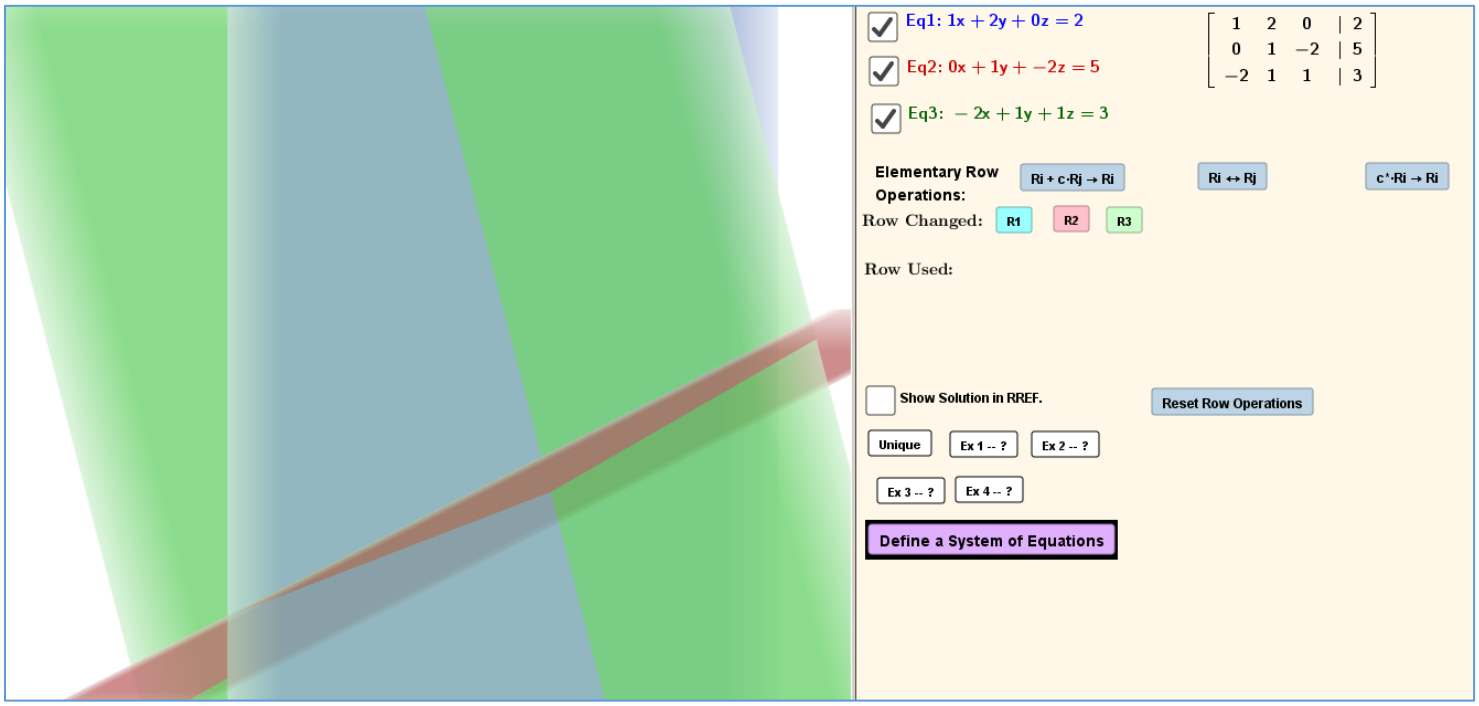


Create and Solve your own system of equations. Click



On this page any system of equations can be created. Once it is chosen then it can be solved using the process on the previous page.


Eq1: 1x + 2y + 0z = 2
Eq2: 0x + 1y + −2z = 5
Eq3: −2x + 1y + 1z = 3
Determine Solution from RREF (reduced row echelon form) obtained by Gauss-Jordan Elimination.


Eq1: 1x + 2y + 0z = 2
Eq2: 0x + 1y + −2z = 5
Eq3: −2x + 1y + 1z = 3
1 2 0 | 2
0 1 −2 | 5
−2 1 1 | 3
Elementary Row Operations:
Ri + c·Rj → Ri
Ri ↔ Rj
c*·Ri → Ri
Row Changed:
R1
R2
R3
Row Used:
Show Solution in RREF.
Reset Row Operations
Unique
Ex 1 -- ?
Ex 2 -- ?
Ex 3 -- ?
Ex 4 -- ?
Define a System of Equations

Transforming Linear Algebra Education with GeoGebra Applets
NSF TUES Grant Award ID: 1141045

Student Activity **Solving Systems of Three Linear Equations in Three Unknowns**

Learning Goals

- Use the Gaussian Elimination applet to solve a system step by step
- Make observations about the related geometric process and relationships

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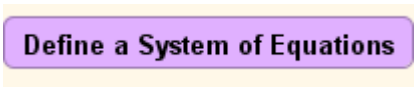
Task **For each system below, use the Gaussian Elimination applet to solve**

Open the Gauss-Jordan applet and click on the button to define a system of three linear equations



to define your selected system of equations. After you are finished, you can go to the main screen to solve the system step by step, using row operations until you have obtained reduced row echelon form.

The graph of each linear equation in three variables can be viewed as a plane in space. Notice that the examples below have variables named x_1, x_2, x_3 , but these can easily be translated into our usual 3-space variable names *x, y,* and *z* .

Before you do any row operations, turn on the visual component for each equation, each pair of equations, and finally all three. Can you make a prediction about the solution?

As you do each row operation, you are replacing one equation with another, keeping the solution to the system unchanged. What can you say about the geometric view of each new equation?

After you have completed this process for several systems, make notes about their differences, both algebraic and graphic. We would like to compare them to each other.

Example systems are on the next page.

Try these:

1. $\begin{aligned} x_1 + 3x_2 + 2x_3 &= 4 \\ 3x_1 + 3x_2 + 2x_3 &= 2 \\ x_2 + x_3 &= 5 \end{aligned}$	2. $\begin{aligned} 2x_1 + 3x_2 + x_3 &= 1 \\ x_1 - x_2 + x_3 &= -2 \\ 2x_1 - 2x_2 + 2x_3 &= -4 \end{aligned}$
3. $\begin{aligned} x_1 &= 0 \\ 3x_2 &= 6 \\ x_1 - x_2 &= 4 \end{aligned}$	4. $\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 3 \end{aligned}$
5. $\begin{aligned} -x_1 + 4x_2 + x_3 &= 0 \\ x_1 + 9x_2 - 2x_3 &= 1 \\ 6x_1 + 4x_2 - 8x_3 &= 0 \end{aligned}$	6. $\begin{aligned} x_1 + x_2 + 3x_3 &= 6 \\ 2x_1 - x_2 + 4x_3 &= 2 \\ -2x_1 - 2x_2 - 6x_3 &= 1 \end{aligned}$
7. $\begin{aligned} 2x_1 + 3x_2 + x_3 &= 2 \\ 4x_1 + 6x_2 + 2x_3 &= 4 \\ -6x_1 - 9x_2 - 3x_3 &= -6 \end{aligned}$	8. $\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ 2x_1 + 2x_2 + x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 3 \end{aligned}$
9. $\begin{aligned} x_1 - 3x_2 + 2x_3 &= -7 \\ 4\,x_1 + x_2 - x_3 &= -5 \\ 6x_1 - 5x_2 + 3x_3 &= -19 \end{aligned}$	10. $\begin{aligned} x_1 + x_2 + x_3 &= 5 \\ x_1 + x_2 - 4x_3 &= 10 \\ -4x_1 + x_2 + x_3 &= 0 \end{aligned}$