# NUMBERS AND SEQUENCES

1. Lemma is an auxiliary result used for proving an important theorem. It is usually considered as a mini theorem
2. Euclid's Division Lemma

   Let $a$ and $b(a < b)$be any two positive integers. Then, their exist unique integers $q$ and $r$such that $a = b\,q + r\,; 0 \le r < b$
3. Generalised form of Euclid's Division Lemma

   If $a$ and $b$ are any two integers then, their exist unique integers $q$ and $r$ such that $a = b\,q + r\,;\quad 0 \le r < |b|$
4. Euclid's Division Algorithm

   To find HCF of two positive integer $a$ and $b$

   Step :1 : Using Euclid's Division Lemma $a = b\,q + r$

   If $r = 0$ the $b$ is the HCF

   Step: 2: Otherwise applying Euclid's Division Lemma $b$ by $r$ to get

   $$b = r\,q_1 + r_1\,;\quad 0 \le r_1 < r$$

   Step:3: If $r_1 = 0$ then $r$is the HCF

   Step:4: Otherwise using Euclid's Division Lemma, repeat the process until to get the remainder zero. The corresponding divisor is the HCF.
5. Another method of finding HCF of two given positive integers

   Step :1 : From the given numbers, subtract the smaller from the larger number

   Step :2 : From the remaining numbers subtract smaller from the larger

   Step :3 : Repeat the subtraction process by subtracting smaller from the larger

   Step :4 : Stop the process, when the numbers become equal

   Step :5 : The number representing equal numbers obtained in Step :4 will be the HCF of the given numbers.
6. Two positive integers are said to be relatively prime or co prime if their HCF is $1$
7. Fundamental Theorem of Arithmetic: Every composite number can be written uniquely as the product of power of primes.
8. When a positive integer is divided by $n$, then the possible remainders are $0, 1, 2, \ldots\ n - 1$
9. Two integers $a$ and $b$ are congruent modula $m$, i.e. $a \equiv b$

$(mod\ m)$, if they leave the same remainder when divided by $m$.

10.$The\ general\ form\ of\ an$ A.P. $a,\ a+d,\ a+2d, \ldots$

11.The difference between two consecutive terms of an AP is always constant. That constant value is called the common difference.

12.If there are finite numbers of terms in an AP then it is called Finite AP

13.If there are infinitely many terms in an AP then it is called Infinite AP.

14.$n^{th}$ term of an AP $\quad t_n = a + (n-1)d$

15.An AP having a common difference of zero is called a constant AP

16.The number of terms of an AP $\quad n = \frac{l-a}{d} + 1$

$a -$ first term $\quad l -$ last term $\quad d -$common difference

17. If every term of an AP is added or subtracted by a constant then the resulting sequence is also an AP

18.If every term of an AP is multiplied or divided by non-zero number then the resulting sequence is also an AP

19.The three consecutive terms of an A P $\ a-d,\ a, and\ \ a+d$

20.The four consecutive terms of an A P $\ a-3d,\ \ a-d,\ a+d, and\ \ a+3d$

21.Three non-zero numbers $a, b, c$ are in AP if and only if $2b = a + c$

22.Sum to $n$ terms of an AP

(i) $S_n = \frac{n}{2}[2a + (n-1)d]$

$a -$ first term $\quad d -$common difference

(ii) $S_n = \frac{n}{2}[a + l]$

$a -$ first term $\quad l -$ last term

23.$The\ general\ form\ of\ a$ G.P. $a,\ a\,r,\ a\,r^2, \ldots$

24.The general term of a G.P. $t_n = a\,r^{n-1}$

$a -$ first term $\quad r -$common ratio

25.Three consecutive terms of a GP $\ \frac{a}{r},\ a,\ a\,r$

26.Four consecutive terms of a GP $\ \frac{a}{r^3}, \frac{a}{r},\ \ a\,r,\ a\,r^3$

27.When each term of a GP is multiplied or divided by a nonzero constant then the resulting sequence is also a GP

28.Sum to $n$ terms of an G P

(i) $S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1;\ \ r > 1$

(ii) $S_n = \frac{a(1-r^n)}{1-r}$ ; $r < 1$

(iii) $S_n = na, \quad r = 1$

29.Sum to infinite terms of an G P $a + a\,r + a\,r^2 + a\,r^3 + \cdots$

$$S_n = \frac{a}{1-r} \;;\; -1 < r < 1$$

30.Sum of first $n$ natural numbers

$$1 + 2 + 3 + \cdots + n = \sum n = \frac{n(n+1)}{2}$$

31.Sum of first $n$ odd natural numbers

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

32.Sum of the squares of first $n$ natural numbers

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

33.Sum of the cubes of first $n$ natural numbers

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$


Prepared by

**K.G.RANGARAJAN M.Sc, B.Ed.,**

***SRIHARI MATHEMATICS ACADEMY***

(TUITION CUM COACHING CENTER),

2/276-G, K.G.NAGAR, KALANGAL(P.O), (VIA) SULUR (T.K),

COIMBATORE(D.T) – 641402

MOBILE NO: 9944196663 E-mail: rangarajankg@gmail.com


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