1.3 1-10 → Solución geogebra.

1.4 Método de variables separables

Ejercicios 5, 12, 17, 18, 25, 27

5. $2\sqrt{x}\,\dfrac{dy}{dx} = \sqrt{1-y^2}$

Separables → producto o entre división funciones

$\dfrac{dy}{dx} = \sqrt{1-y^2} \cdot \dfrac{1}{2\sqrt{x}}$

$\rightarrow dy = \sqrt{1-y^2} \cdot \dfrac{1}{2\sqrt{x}} \cdot dx$

$\rightarrow \displaystyle\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{1}{2\sqrt{x}} \cdot dx$

$\rightarrow \text{arsen}\, y = \sqrt{x} + C$

$\hookrightarrow \text{sen}^{-1} y = \sqrt{x} + C$

$y(x) = \text{sen}(\sqrt{x} + C)$

→ una Solución general de forma explícita

$\displaystyle\int \frac{1}{2\sqrt{x}}\, dx$

$\rightarrow \displaystyle\int \frac{1}{2} x^{-1/2}\, dx$

$\rightarrow \dfrac{1}{\cancel{2}} \cdot \dfrac{x^{1/2}}{1/\cancel{2}} + C$

$= \sqrt{x} + C$

integral

$\displaystyle\int \frac{1}{\sqrt{1-u^2}}\, du = \text{arsen}\, u + C$

$\rightarrow \displaystyle\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{1}{2\sqrt{x}}\, dx$

$= \text{arsen}\, y = \sqrt{x} + C$

12. $yy' = x(y^2+1)$

$$y\frac{dy}{dx} = x(y^2+1)$$

$$\frac{dy}{dx} = \frac{x\cdot(y^2+1)}{y}$$

$$dy = x\cdot\frac{(y^2+1)}{y}\cdot dx$$

$$\int\frac{y}{y^2+1}\,dy = \int x\,dx$$

$$\frac{1}{2}\ln(y^2+1) = \frac{x^2}{2}+C$$

$$\ln(y^2+1) = x^2+c_2$$

$$\int\frac{y}{y^2+1}\,dy \qquad U = y^2+1$$

$$du = 2y\,dy$$

$$\int\frac{1}{y^2+1}\,y\,dy \qquad \frac{du}{2} = y\,dy$$

$$\int\frac{1}{U}\cdot\frac{du}{2}$$

$$\frac{1}{2}\int\frac{1}{U}\,du$$

$$\frac{1}{2}\ln U + C_1$$

$$\frac{1}{2}\ln(y^2+1)+C$$

$$e^{\ln(y^2+1)} = e^{x^2+c_2}$$

$$y^2+1 = e^{x^2}\cdot e^{c_2}$$

$$y^2+1 = Ae^{x^2}$$

→ una solución general de forma implícita

17 $y' = 1 + x + y + xy$

$$\frac{dy}{dx} = 1 + x + y + xy$$

$$\frac{dy}{dx} = (1+x)(y+xy)$$

$$\frac{dy}{dx} = (1+x) + y(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y)$$

$$dy = (1+x)(1+y)\cdot dx$$

$$\int \frac{dy}{(1+y)} = \int (1+x)\cdot dx$$

$$\ln(1+y) = x + \frac{x^2}{2} + C$$

$$e^{\ln(1+y)} = e^{x+\frac{1}{2}x^2+C}$$

$$e^{\ln(1+y)} = e^{x+\frac{1}{2}x^2} \cdot e^{C}$$

$$1+y = Ae^{\frac{1}{2}x^2+x}$$

$$y(x) = Ae^{\frac{1}{2}x^2+x} - 1$$

Solución general de facción explícita

18. $x^2 y' = 1 - x^2 + y^2 - x^2 y^2$

$$x^2 \cdot \frac{dy}{dx} = (1-x^2) + (y^2 - x^2y^2)$$

$$x^2 \cdot \frac{dy}{dx} = (1-x^2) + y^2(1-x^2)$$

$$x^2 \cdot \frac{dy}{dx} = (1-x^2)(1+y^2)$$

$$\frac{dy}{dx} = \frac{1-x^2}{x^2} \cdot (1+y^2)$$

$$dy = \frac{1-x^2}{x^2} \cdot (1+y^2) \cdot dx$$

$$\int \frac{dy}{1+y^2} = \int \frac{1-x^2}{x^2} \cdot dx$$

$$\int \frac{dy}{1+y^2} = \int \frac{1}{x^2} - 1\, dx$$

en este caso $u = y$

→ integral por C.1

$$\int \frac{1}{1+u^2}\, du \rightarrow \arctan u + C$$

$$\arctan y = -\frac{1}{x} - x + C$$

$$y(x) = \tan\left(C - x - \frac{1}{x}\right)$$

una solución general forma explícita