

## Sección 1.4 Método de variables separables

5)

$$2 \sqrt{x} \frac{dy}{dx} = \sqrt{1-y^2}$$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{\sqrt{x}}{2\sqrt{x}}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{2\sqrt{x}}$$

$$\boxed{\arcsin y = \sqrt{x} + C} \rightarrow \text{implícita (y no despejada)}$$

$$\hookrightarrow y = \sin(\sqrt{x} + C) \rightarrow \text{explícita}$$

↓  
solución general forma implícita

12)  $yy' = x(y^2 + 1)$

$$\frac{dy}{dx} = \frac{x(y^2 + 1)}{y}$$

$$\frac{dy}{y} = \frac{x \cdot (y^2 + 1)}{y} \cdot dx$$

$$dy = \frac{x \cdot (y^2 + 1)}{y} dx$$

$$\int \frac{y dy}{(y^2 + 1)} = \int x \cdot dx$$

$$\frac{1}{2} \ln(y^2 + 1) = \frac{x^2}{2} + C$$

$$\ln(y^2 + 1) = x^2 + C_2$$

$$2C \rightarrow C_2$$

$$e^{\ln(y^2 + 1)} = e^{x^2} e^{C_2}$$

$$y^2 + 1 = A e^{x^2} \rightarrow \text{sol. general forma implícita}$$

$$y^2 = A e^{x^2} - 1$$

$$17) y' = 1 + x + y + xy$$

$$\frac{dy}{dx} = 1 + x + y + xy$$

$$\frac{dy}{dx} = (1+x)(y+1)$$

$$\frac{dy}{dx} = (1+x) + y(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y)$$

$$\int \frac{dy}{1+y} = \int (1+x) dx$$

$$\ln(1+y) = x + \frac{x^2}{2} + C \rightarrow e^{\ln(1+y)} = e^{x + \frac{1}{2}x^2 + C}$$

$$1+y = e^{x + \frac{1}{2}x^2} \cdot e^C$$

$$y = e^{x + \frac{1}{2}x^2} \cdot A - 1$$

↓  
solucion general explicita

$$18) \frac{dy}{dx} = 1 - x^2 + y^2 - x^2 y^2$$

$$x^2 \frac{dy}{dx} = (1-x^2) + (y^2 - x^2 y^2)$$

$$x^2 \frac{dy}{dx} = (1-x^2) + y^2(1-x^2)$$

$$x^2 \frac{dy}{dx} = (1-x^2)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int \frac{(1-x^2) dx}{x^2}$$

$$\arctan y = \int \frac{1}{x^2} - 1 dx$$

$$\arctan y = -\frac{1}{x} - x + C$$

$$y = \tan\left(-x - \frac{1}{x}\right)$$

$$25) \quad x \frac{dy}{dx} - y = 2x^2 y \quad y(1) = \underline{1}$$

$$x \frac{dy}{dx} = 2x^2 y + y$$

$$x \frac{dy}{dx} = y(2x^2 + 1)$$

$$\frac{dy}{y} = \frac{(2x^2 + 1)}{x} dx$$

$$dy = \frac{y(2x^2 + 1)}{x} dx$$

$$\int \frac{dy}{y} = \int \frac{(2x^2 + 1)}{x} dx$$

$$\int \frac{dy}{y} = \int \left( 2x + \frac{1}{x} \right) dx$$

$$= \ln y = x^2 + \ln x + C$$

$$e^{\ln y} = e^{x^2} e^{\ln x} e^C$$

$$y = Ax e^{x^2}$$

Wanda  $y(1) = \frac{1}{2}$

$$\frac{1}{2} = A(1)e^{(1)^2}$$

$$\frac{1}{2} = Ae$$

$$\frac{1}{2e} = A$$

B//  $y(x) = \frac{1}{2e} x e^{x^2}$