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Related Rates

By: Arq. Monica M. Paniagua, Ing. Zied Najjar, Lic. Lucy Solis, Lic Carmela de la fuente

Name Faisal Fernández del Bosque ID # A01570170 Date

Related rates

1. A spherical balloon is being filled at a rate of 50in³/sec, at what rate does the radius increase when the radius is 5in?

$v = \frac{4}{3}\pi r^3$ $\quad 50 = 4\pi(5)^2\frac{dr}{dt}$ $\quad \frac{dr}{dt} = \frac{50}{4\pi(25)} = \frac{50}{100\pi} = \frac{1}{2\pi}$

$\frac{dv}{dt} = \frac{4}{3}(3)\pi r^2$ $\quad 50 = 4\pi\, 25\frac{dr}{dt}$

2. The area of a circle is increasing at a rate of 20in²/min. Find the rate at which the radius is increasing when the radius is 4in.

$\frac{dA}{dt} = \pi 2r\frac{dr}{dt}$ $\quad A = \pi r^2$ $\quad \frac{dA}{dt} = 20in^2/min$ $\quad \frac{dr}{dt} = \frac{5}{2\pi}\ in/min$

$20 = \pi 2(4)\frac{dr}{dt}$

3. A stone is thrown into a lake and a circular ripple moves out at a constant rate of 0.5 meters/sec. Find the rate at which the circle's area is increasing at r = 0.4 meters.

$\frac{dA}{dt} = \pi 2(0.4)(0.5)$ $\quad \frac{dr}{dt} = 0.5m/s$ $\quad \frac{dA}{dt} = \pi 2r\frac{dr}{dt}$

$\frac{dA}{dt} = 0.4\pi\ m^2/s$ $\quad r = 0.4m$

4. Air is being pumped into a spherical balloon making the radius change at a constant rate of 0.5cm/sec. Find the rate of change of the volume and the rate of change of the surface area when the radius is 10cm $(V = \frac{4}{3}\pi r^3,\ A = 4\pi r^2)$

$\frac{dv}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$ $\quad \frac{dr}{dt} = 0.5cm/sec$ $\quad \frac{dA}{dt} = 4\pi 2r\frac{dr}{dt}$

$\frac{dv}{dt} = 4\pi(10)^2(0.5)$ $\quad r = 10cm$

$\frac{dv}{dt} = 200\pi\ cm/s$ $\quad v = \frac{4}{3}\pi r^3$ $\quad A = 4\pi r^2$

5. A cone is increasing in size as time goes by in such a way that the volume is changing at a constant rate of 75cm³/min. The height is twice the radius. Determine the rate of change of the height, when the height is 5cm. $(V = \frac{1}{3}\pi r^2 h)$

$\frac{dv}{dt} = \frac{\pi}{4}3h^2\frac{dr}{dt}$ $\quad \frac{dv}{dt} = 75cm^3/min$ $\quad h\ 5cm$

$75 = \frac{\pi}{4}\pi(5)^2\frac{dr}{dt}$ $\quad v = \frac{\pi h^3}{r^2}$ $\quad r = 2.5cm$

$h = ?$ $\quad r = h/2$

90

Rules of Differentiation- Logarithmic Differentiation

By: Ing. Zied Najjar

Name Faisal Fernández ID A01570170

Date

For this method of differentiation, you need to recall the 3 logarithmic properties

$\ln(a^n) = n\ln(a)$

$\ln(ab) = \ln(a) + \ln(b)$

$\ln(\frac{a}{b}) = \ln(a) - \ln(b)$

example

$\ln(1-5x)^3 = 3\ln(1-5x)$

$\ln(5x(2x-1)^7) = \ln(5x) + \ln(2x-1)^7 = \ln(5x) + 7\ln(2x-1)$

$\ln\left(\frac{x^3}{\sqrt{4x-1}}\right) = \ln(x^3) - \ln(4x-1)^{1/2} = 3\ln(x) - \frac{1}{2}\ln(4x-1)$

Example:

Use logarithmic differentiation to find the derivative of:

Applications of derivatives

Problems involving position, velocity and acceleration

67

Name Faisal Fernández ID A01570170 Date

Consider each of the following situations and answer clearly. Remember to use the appropriate mathematical notation and to frame your final answer.

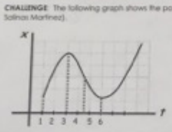
1. An object is moving along a straight line, and its position (in meters) is given by the function $s(t)=80t-t^2$. Determine
   a) The velocity of the object when t = 2 sec 76m/s
   b) The acceleration when t = 3 sec 2 m/s²
   c) The time when the velocity is zero and the position of the object at that time 40s 1600m
2. An object is moving along a straight line, and its position (in meters) is given by the function $s(t)=3t+\frac{48}{t+1}$

   Determine
   a) The velocity of the object when t = 2 sec 2.33 m/s
   b) The acceleration when t = 2 sec 3.5 m/s²
   c) The time when the velocity is zero and the position of the object at that time 3s 21m
3. A dynamite charge blows a rock up with a velocity of 160 feet/sec. The height of the rock is given by $h(t)=160t-16t^2$ where "h" is measured in feet and "t" in seconds. Find
   a) The equation that gives the velocity of the rock at any time 160 - 32t
   b) The time when the velocity is zero 5s
   c) The height of the rock when the velocity is zero (maximum height) 400m
   d) The times (on the way up and on the way down) when the height is 256 feet 11.37 and -1.37
   e) The velocities of the rock when the height is 256 feet
   f) The equation that gives the acceleration of the rock at any time
   g) How long does it take the rock to fall back down?
4. A baseball is thrown upward while being in the moon (hypothetically), with an initial velocity of 24 meters/second. The height of the ball is given by $s=24t-0.8t^2$
   a) Find the equations of velocity and acceleration at any time v = 24 - 1.6t a = -1.6
   b) How long does it take the ball to reach its maximum height? 15 s
   c) Find the maximum height of the ball 180m
   d) How long was the ball in the air? 30s
5. The position of an object is given by $S(t)=t^3-6t^2+9t$ where "t" is measured in seconds and "s" in meters.
   a) Find the equations of velocity and acceleration as a function of time $v = 3t^2-12t+9$ $a = 6t-12$
   b) Find the time when the velocity is zero 3s
   c) Find the acceleration when the velocity is zero a = 6m/s²
   d) Find the time when the acceleration is zero and then give the velocity at that time. t = 2s
6. The height of a certain tree (starting from being 1 year old) is modeled by $H(t)=5\sqrt{t^3}+2t^2+10$, where height is measured in cm and time in years

   Find:
   a) The height of the tree in its 5th year (hint t=4) 82cm
   b) The function that models the rate of change of its height $15t^{1/2}\sqrt{t}+4t$
   c) The rate of change when t=4 31 cm
   d) The rate of change when t=9 38.5 m
   e) When is the tree growing faster? at t=4 or t=9 years? Why? t=4

   $\frac{15t^2}{2\sqrt{t^3}}+4t$

CHALLENGE: The following graph shows the position of a particle that moves along a straight line (author: Lic. Norma Patricia Salinas Martinez).



a) In which interval or intervals is the velocity of the particle positive?
b) In which interval or intervals is the velocity of the particle negative?
c) In which interval or intervals of time is the position increasing faster?
d) In which interval or intervals of time is the position increasing slower?
e) In which interval or intervals of time is the position decreasing faster?
f) In which interval or intervals of time is the position decreasing slower?
g) In which interval or intervals of time is the velocity increasing?
h) In which interval or intervals of time is the velocity decreasing?

Prepa Tec Calculus I 3rd partial 70 Quiz # 2A

Name Faisal Fernández del Bosque Mat. A01570170

Determine if true or false for each of the following statements. (5 points each)

1. T The function $f(x)=x^2-4$ has a relative maximum at (0, -4)
2. F If "a" is a critical point of the function f(x) that is continuous, and if f'(x) < 0 at (-∞, a) and f'(x) > 0 at (a, ∞), then, f(x) has a relative minimum at (a, f(a)).
3. T Let f be a function whose second derivative exists on an open interval, if f''(x) >0 for all x in that interval, then the graph of f is concave upward on that interval.
4. F The function $f(x)=-3x^3-3x^2+14$ has only one critical point.

$-9x^2-6x$

$-18x-6$

Choose the right answer (10 points each)

1. (D) If (c, f(c)) is a critical point, then
   A) f(c)=0 B) f'(c)<0 C) f'(c)>0 D) f'(c)=0
2. (A) According to the second derivative test if f''(c) >0, then
   A) f(c) is concave downward.
   B) f(c) is relative maximum.
   C) f(c) is a critical point.
   D) f(c) is a relative minimum.
3. (A) The function $f(x)=20x-x^2$ has a critical point:
   A) x=10 B) x=-10 C) x=0 D) x=1

   $20-2x$

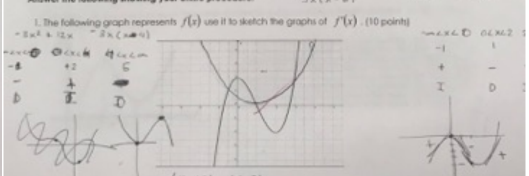
   $2(10-x)$
4. (B) If $(f''(x)>0)$ then f(x) is:
   A) concave upward B) concave downward C) decreasing D) increasing
5. (B) It can be determined if the curve of y=f(x) has a change of concavity:
   A) Critical point B) Inflection point C) x-intersect D) y-intercept
6. (B) The function $y=-x^3+6x^2$ has a relative minimum at:
   A) (4,-32) B) (0, 0) C) (4, 32) D) (6, 0)
7. (C) The function $y=x^3-3x^2$ has a relative maximum at:
   A) (1,-2) B) (3, 0) C) (0, 0) D) (2,-4)

   $3x^2-6x$ $\quad x=2$ (2, -4)

   $3x(x-2)$ $\quad x=0$ (0, 0)

Answer the following showing your entire procedure.

1. The following graph represents $f(x)$ use it to sketch the graphs of $f'(x)$. (10 points)



(4, 32) (0,0)

Prepa Tec
Calculus I 3rd partial
Quiz # 1A

80

Name Faisal Fernánda au Bosque Mat. A01570110

I. Determine if true or false for each of the following statements (10 points each)

1. F The second derivative of $y = 2e^{x^3}$ is $\frac{d^2y}{dx^2} = 6xe^{x^3}(2+3x^3)$

2. F The derivative of $6x - 4x^2y = 2y^2 + 1$ is $\frac{dy}{dx} = \frac{-6}{8x+4y}$

3. T The derivative of $y = x^{2x}$ is $y' = 2x^{2x}(\ln(x)+1)$

4. T A spherical balloon is being inflated with a gas at a rate of 6 cm³ per second. Then the rate at which its radius is changing when its radius measures 8 cm is $\frac{dr}{dt} = \frac{3\pi}{128}\left[\frac{cm}{sec}\right]$. (Hint: $V = \frac{4}{3}\pi r^3$)

$\frac{dv}{dt} = 6 \quad r = 8$

$6 - 4\pi(8)^2\frac{dr}{dt}$

$\frac{6}{256\pi} = \frac{dr}{dt}$

$6x - 4x^2y = 2y^2 + 1$

II. Answer the following problem. (10 points each letter)

A dynamite charge blows a rock up with a velocity of 160 ft/s. The height of the rock is given by the function $h(t) = 160t - 16t^2$ where "h" is measured in feet and "t" in seconds. Find the following:

$256 = 160t - 16t^2$

$-16t^2 + 160t - 256$

a) The equation that gives the velocity of the rock at any time.

$v(t) = 160 - 32t$

b) The time when velocity is zero (that is the time to reach the maximum height)

$0 = 160 - 32t \qquad \frac{-160}{-32} = t \qquad t = (5)$

c) The maximum height of the rock (that is when velocity is zero)

$h(5) = 160(5) - 16(5)^2$

$h(5) = 800 - 400$

$\boxed{h(5) = 400m}$

d) The times (on the way up and on the way down) when the height is at 256 feet.

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \frac{-160 \pm \sqrt{160^2 - 4(-16)(-256)}}{2(-16)} \qquad \frac{-160 \pm \sqrt{9216}}{-32} \qquad \frac{-160 \pm 96}{-32}$

$t^1 = 2s \qquad t^2 = 8s$

e) The velocities of the rock when the height is 256 feet.

$v(2) = 160 - 32(2) \qquad v(8) = 160 - 32(8)$

$\boxed{v(2) = 96\ m/s} \qquad \boxed{v(8) = -96\ m/s}$

f) The equation that gives the acceleration of the rock at any time.

$a(t) = -32$

Second Quiz

① $x^4 - 4 \qquad 4x^3$ F- because its minimum

$-\infty < x < 0$	$0 < x < \infty$
-1	1
-4	4
−	+
∩	U

② is a relative minimum

④ concave upward

First Quiz

① $2e^{x^3}$

$y' = 6x^2e^{x^3} \qquad 12xe^{x^3} + 6x^2(3x^2e^{x^3})$

$6x^2 \quad 12x \qquad 12xe^{x^3} + 18x^4e^{x^3}$

$e^{x^3} \quad 3x^2e^{x^3} \qquad 6xe^{x^3}(2 + 3x^3)$

④ $\frac{4}{3}\pi 3r^2\frac{dr}{dt}$

$6 = 4\pi(8)^2\frac{dr}{dt}$

$\frac{6}{256\pi} = \frac{dr}{dt} \qquad$ F