Домаћи рад Матија Копривица 3-3

1210. а)

$$\lim_{n\to\infty}\frac{(n+1)^3-(n-1)^3}{(n+1)^2+(n-1)^2} = ?$$

$$\lim_{n\to\infty}\frac{(n+1)^3-(n-1)^3}{(n+1)^2+(n-1)^2} = \lim_{n\to\infty}\frac{\cancel{n^3}+3n^2\cancel{+3n}+1-(\cancel{n^3}-3n^2\cancel{+3n}-1)}{n^2\cancel{+2n}+1+n^2\cancel{-2n}+1} =$$

$$\lim_{n\to\infty}\frac{6n^2+2\;/:n^2}{2n^2+2\;/:n^2} = \lim_{n\to\infty}\frac{6+\frac{2}{n^2}}{2+\frac{2}{n^2}} = \frac{6}{2} = \boxed{3}$$

в)

$$\lim_{n\to\infty}\left(\frac{2n^2}{2n+3}+\frac{1-3n^2}{3n^2+1}\right) = ?$$

$$\lim_{n\to\infty}\left(\frac{2n^2}{2n+3}+\frac{1-3n^2}{3n^2+1}\right) = \lim_{n\to\infty}\frac{2n^2(3n^2+1)+(1-3n^3)(2n+3)}{(2n+3)(3n^2+1)} =$$

$$\lim_{n\to\infty}\frac{\cancel{6n^4}+2n^2+2n+3\cancel{-6n^4}-9n^3}{6n^3+2n+9n^2+3} = \lim_{n\to\infty}\frac{-9n^3+2n^2+2n+3\;/:n^3}{6n^3+9n^2+2n+3\;/:n^3} =$$

$$\lim_{n\to\infty}\frac{-9+\frac{2}{n}+\frac{2}{n^2}+\frac{3}{n^3}}{6+\frac{9}{n}+\frac{2}{n^2}+\frac{3}{n^3}} = -\frac{9}{6} = \boxed{-\frac{3}{2}}$$

1211. б)

$$\lim_{n\to\infty}\frac{(n+2)!-(n+1)!}{(n+3)!} =?$$

$$\lim_{n\to\infty}\frac{(n+2)!-(n+1)!}{(n+3)!} = \lim_{n\to\infty}\frac{(n+2)(n+1)!-(n+1)!\;/:(n+1)!}{(n+3)(n+2)(n+1)!\;/:(n+1)!} =$$

$$\lim_{n\to\infty}\frac{n+2-1}{(n+3)(n+2)} = \lim_{n\to\infty}\frac{n+1}{(n+3)(n+2)} = \lim_{n\to\infty}\frac{n}{n\cdot n} = \lim_{n\to\infty}\frac{1}{n} = 0$$

1213. ђ)

$$\lim_{n\to\infty}\left(\frac{1+5+9+\cdots+(4n-3)}{2(n+1)}-n\right)=?$$

$$\lim_{n\to\infty}\left(\frac{\overbrace{1+5+9+\cdots+(4n-3)}^{S_n}}{2(n+1)}-n\right)$$

$$=\lim_{n\to\infty}\left(\frac{n(2n-1)}{2(n+1)}-n\right)$$

$$=\lim_{n\to\infty}\frac{2n^2-n-2n(n+1)}{2(n+1)}=\lim_{n\to\infty}\frac{-3n}{2n+2}$$

$$=\lim_{n\to\infty}\frac{-2n}{6n}=\boxed{-\frac{3}{2}}$$

$a_1=1;\ d=4;\quad a_n=4n-3$

$$S_n=\frac{n}{2}(2a_1+(n-1)d)=\frac{n}{2}(2\cdot 1+(n-1)\cdot 4)$$

$$=n(2n-1)$$