

# ZADATCI ZA VJEŽBANJE – ISPIT VEKTORI I PRAVCI

Lara Vučić, 3.c

Vježba za ispit  
ZADATCI

1. Dan je paralelogram ABCD. Točke P i Q polovišta su stranica BC i CD. Vektori  $\vec{AP}$ ,  $\vec{AQ}$  i  $\vec{PQ}$  rastavi u komponente po vektorima  $\vec{e}_1 = \vec{AB}$  i  $\vec{e}_2 = \vec{AD}$ .

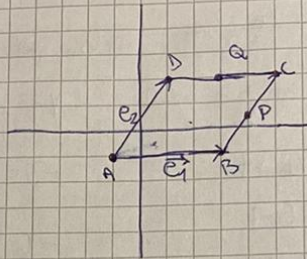
Zadane su točke:

$$A(-1, -1)$$

$$B(3, -1)$$

$$C(5, 2)$$

$$D=? \quad D(x, y)$$



$$\vec{AD} + \vec{DC} = \vec{AC} = \vec{AB} + \vec{BC}$$

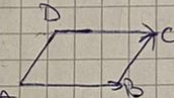
$$\vec{AD} = \vec{BC}$$

$$(b_x - a_x)\vec{i} + (b_y - a_y)\vec{j} = \vec{AD}$$

$$(3 - (-1))\vec{i} + (-1 - (-1))\vec{j} = \vec{AD}$$

$$4\vec{i} + 0\vec{j} = \vec{AD}$$

$$\vec{AD} = 4\vec{i}$$



$$(c_x - d_x)\vec{i} + (c_y - d_y)\vec{j} = \vec{DC}$$

$$(5 - x)\vec{i} + (2 - y)\vec{j} = \vec{DC}$$

$$\vec{AD} = \vec{DC}$$

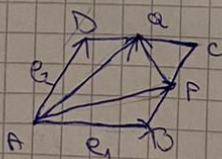
$$4\vec{i} = (5 - x)\vec{i} + (2 - y)\vec{j}$$

$$4 = 5 - x \quad 0 = 2 - y$$

$$x = 1$$

$$y = 2$$

$$\boxed{D(1, 2)}$$



$$\vec{AP} = \vec{AB} + \vec{BP}$$

$$\vec{AP} = \vec{e}_1 + \frac{1}{2}\vec{e}_2$$

$$\vec{AQ} = \vec{AD} + \vec{DQ}$$

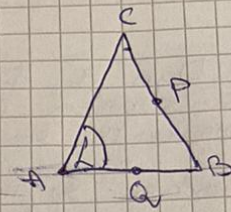
$$\vec{AQ} = \vec{e}_2 + \frac{1}{2}\vec{e}_1$$

$$\vec{PQ} = -\vec{AP} + \vec{AQ}$$

$$\vec{PQ} = -\vec{e}_1 - \frac{1}{2}\vec{e}_2 + \vec{e}_2 + \frac{1}{2}\vec{e}_1$$

$$\vec{PQ} = -\frac{1}{2}\vec{e}_1 + \frac{1}{2}\vec{e}_2$$

2. Točke  $P(5,1)$  i  $Q(3,3)$  polovištan su stranica  $BC$  i  $AB$  trokuta  $ABC$ . Ako je  $B(7,-4)$ , odredi koordinate ostalih dva vrhova trokuta.



a) odredi koordinate težišta trokuta  $ABC$ ,

b) izračunaj površinu trokuta,

c) odredi kut  $\alpha$ .

a)

$$x_Q = \frac{x_A + x_B}{2}$$

$$B(7, -4)$$

$$A(x_A, y_A)$$

$$3 = \frac{x_A + 7}{2} \quad | \cdot 2$$

$$y_Q = \frac{y_A + y_B}{2}$$

$$6 = x_A + 7$$

$$-3 = \frac{y_A + (-4)}{2} \quad | \cdot 2$$

$$x_A = 6 - 7$$

$$-6 = y_A - 4$$

$$x_A = -1$$

$$y_A = -2$$

$$A(-1, -2)$$

$$B(7, -4)$$

$$x_P = \frac{x_B + x_C}{2}$$

$$y_P = \frac{y_B + y_C}{2}$$

$$P(5, 1)$$

$$5 = \frac{7 + x_C}{2} \quad | \cdot 2$$

$$1 = \frac{-4 + y_C}{2} \quad | \cdot 2$$

$$C(x_C, y_C)$$

$$10 = 7 + x_C$$

$$2 = -4 + y_C$$

$$x_C = 3$$

$$y_C = 6$$

$$C(3, 6)$$

$$T(x_T, y_T)$$

$$x_T = \frac{x_A + x_B + x_C}{3}$$

$$y_T = \frac{y_A + y_B + y_C}{3}$$

$$x_T = \frac{-1 + 7 + 3}{3}$$

$$y_T = \frac{-2 - 4 + 6}{3}$$

$$x_T = 3$$

$$y_T = 0$$

$$T(3, 0)$$

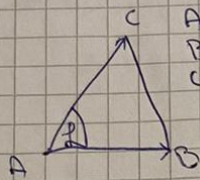


$$b) P_{\Delta} = \frac{1}{2} (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$$

$$P_{\Delta} = \frac{1}{2} (-1(-4-6) + 7(6-2) + 3(2+4))$$

$$P_{\Delta} = 28$$

c)



$$A(1,2) \quad B(7,1) \quad C(3,6) \quad \vec{AB} \cdot \vec{AC} = |\vec{AB}| \cdot |\vec{AC}| \cos \alpha$$

$$(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} = \vec{AB}$$

$$(7+1)\vec{i} + (-4-2)\vec{j} = \vec{AB}$$

$$\boxed{\vec{AB} = 8\vec{i} - 6\vec{j}}$$

$$(x_C - x_A)\vec{i} + (y_C - y_A)\vec{j} = \vec{AC}$$

$$(3+1)\vec{i} + (6-2)\vec{j} = \vec{AC}$$

$$\boxed{\vec{AC} = 4\vec{i} + 4\vec{j}}$$

$$|\vec{AB}| = \sqrt{8^2 + (-6)^2}$$

$$= 10$$

$$|\vec{AC}| = \sqrt{4^2 + 4^2}$$

$$= 4\sqrt{2}$$

$$\cos \alpha = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$$

$$\cos \alpha = \frac{(8 \cdot 4) + (-6 \cdot 4)}{10 \cdot 4\sqrt{2}}$$

$$\cos \alpha = \frac{56}{40\sqrt{2}} = 0.9899$$

$$\boxed{\alpha = 8^\circ 9' 0,47''}$$

3. Napiši zadanu jednačbu u eksplcitnom i segmentnom obliku. Potom nacrтай pravac te jednačbe.

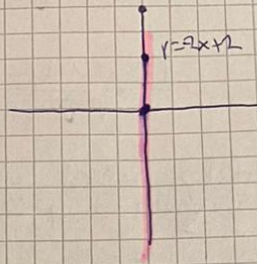
ZADANA JEDNAČBA (u implicitnom obliku)

$$2x + y - 2 = 0$$

$$\text{EKS: } y = -2x + 2$$

$$\text{SEG: } 2x + y = 2 \quad | :2$$

$$\frac{x}{1} + \frac{y}{2} = 1$$



4. Trokut koji zatvaraju pravci  $x + 2y - 3 = 0$ ,  $x + y - 5 = 0$  i  $4x - 2y + 11 = 0$  je pravokutan. Proveri!

$$x + 2y - 3 = 0$$

$$-2y = -x + 3 \quad | :(-2)$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

$$k_1 = \frac{1}{2}$$

$$k_2 = -1$$

$$k_3 = 2$$

$$k_1 \cdot k_2 = ?$$

$$\frac{1}{2} \cdot (-1) = \frac{1}{2} \times$$

$$k_2 \cdot k_3 = ?$$

$$-1 \cdot 2 = -2 \times$$

$$k_3 \cdot k_1 = ?$$

$$2 \cdot \left(\frac{1}{2}\right) = ?$$

$$2 \cdot \left(\frac{1}{2}\right) = 1 \checkmark$$

Da. Pravci zatvaraju trokut koji je pravokutan

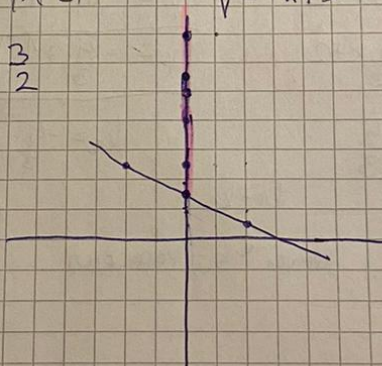
$$x + y - 5 = 0$$

$$y = -x + 5$$

$$4x - 2y + 11 = 0$$

$$-2y = -4x - 11 \quad | :(-2)$$

$$y = \frac{2x + 11}{2}$$





5. Koliko iznosi udaljenost točke  $T(-1, 3)$  od pravca  
 $3x + 4y + 6 = 0$

$$\frac{|Ax + By + C|}{\sqrt{A^2 + B^2}}$$

$$\frac{|3 \cdot (-1) + 4 \cdot 3 + 6|}{\sqrt{3^2 + 4^2}} = \frac{15}{5} = 3$$

6. Koliko iznosi udaljenost između pravaca

$$4x + 3y - 2 = 0 \quad \text{i} \quad 8x + 6y - 1 = 0$$

$$3y = -4x + 2$$

$$6y = -8x + 1$$

$$y = -\frac{4}{3}x + \frac{2}{3}$$

$$y = -\frac{4}{3}x + \frac{1}{6}$$

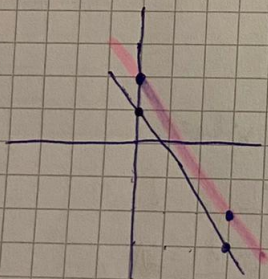
$T(0, \frac{2}{3})$

$$k_1 = -\frac{4}{3}$$

$$k_2 = -\frac{4}{3}$$

$$k_1 = k_2$$

Pravci su paralelni



$$\frac{|Ax + By + C|}{\sqrt{A^2 + B^2}} = \frac{|8 \cdot 0 + 6 \cdot \frac{2}{3} - 1|}{\sqrt{8^2 + 6^2}} = \frac{11}{10}$$