PROJECTILE MOTION *Coconut*

A rifle is fired at a coconut in a tree; but the coconut falls just as the rifle is fired. Will the bullet hit the falling coconut? Take the height of the coconut to be H, above the level of the rifle, which will be zero in the y direction. The horizontal distance is X. We can write the kinematic relations

$$x_c(t) = X \qquad y_c(t) = H - \frac{1}{2} g t^2 \qquad \text{coconut}$$

$$x_b(t) = v_0 \cos(\theta)\, t \qquad y_b(t) = v_0 \sin(\theta)\, t - \frac{1}{2} g t^2 \qquad \text{bullet}$$

Let T be the time of flight to the coconut's horizontal position X; then

$$x_c(T) = X = x_b(T) = v_0 \cos(\theta)\, T \qquad T := \frac{X}{v_0 \cos(\theta)}$$

The y coordinate of the bullet at time T will be

$$y_b(T) = \left[v_0 \sin(\theta) \frac{X}{v_0 \cos(\theta)} - \frac{1}{2} g \left(\frac{X}{v_0 \cos(\theta)} \right)^2 \right] = X \tan(\theta) - \frac{g X^2}{2 \left(v_0 \cos(\theta)\right)^2}$$

and the y of the coconut is

$$y_c(T) = H - \frac{g X^2}{2 \left(v_0 \cos(\theta)\right)^2}$$

If there was a collision, then these y values must be equal, so by inspection

$$X \tan(\theta) = H$$

and this is correct for the geometry of the problem, that is, if the rifle was at an angle θ, aimed at the coconut in the tree at time zero.

We should check that the time required for the bullet to get there is not longer than it takes for the coconut to hit the ground. That time will be

$$0 = H - \frac{1}{2} g T_c^2 \qquad T_c = \sqrt{\frac{2 H}{g}}$$

Next we have the condition that the y-coordinate of the bullet is also just at zero (the ground) at this same time, so that

$$0 = v_0 \sin(\theta)\, T_c - \frac{1}{2} g T_c^2 = v_0 \sin(\theta) \sqrt{\frac{2 H}{g}} - \frac{1}{2} g \left(\sqrt{\frac{2 H}{g}} \right)^2$$

from which we find the condition for collision

$$v_0 \sin(\theta) \geq \sqrt{\frac{g H}{2}}$$