2.5 – A Polynomial Multiplied by a Polynomial

Below is an algebraic expression that prompts polynomial by polynomial multiplication:

$$(x+2)\cdot(x^2+2x)$$

If your current thoughts are: "how the *heck* am I going to do this?!," I promise you that this section is no more difficult than the previous. In fact, it is pretty much the same – and with a new trick, you might actually find it easier!

Repeated Distribution

Repeated Distribution is a textbook approach for carrying out polynomial by polynomial multiplication. This method suggests that we repeat the Distributive Property of Multiplication Over Addition for each term of our 1^{st} factor.

Example 1:

Multiply x + 2 by $x^2 + 2x$.

$$(x + 2)(x^{2} + 2x)$$

$$x(x^{2} + 2x) + 2(x^{2} + 2x)$$

$$\downarrow$$

$$(x \cdot x^{2} + x \cdot 2x) + (2 \cdot x^{2} + 2 \cdot 2x)$$

$$\downarrow$$

$$x^{3} + 2x^{2} + 2x^{2} + 4x$$

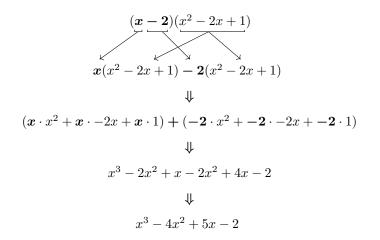
$$\downarrow$$

$$x^{3} + 4x^{2} + 4x$$

Solution: First, we enclose both binomials in parentheses. Now, we distribute the 1^{st} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow x(x^2 + 2x)$, and we distribute the 2^{nd} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow 2(x^2 + 2x)$. Once distributed, we combine like terms.

Example 2:

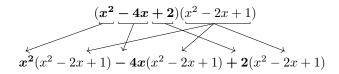
Multiply x - 2 by $x^2 - 2x + 1$.



Solution: First, we enclose both binomials in parentheses. Now, we distribute the 1^{st} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow x(x^2 - 2x + 1)$, and we distribute the 2^{nd} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow -2(x^2 - 2x + 1)$. Note, we must not ignore the subtraction sign. The subtraction sign is included as a negative sign in our 2^{nd} term. When distributing, be sure to distribute the entire term. Once distributed, we combine like terms.

Example 3:

Multiply $x^2 - 4x + 2$ by $x^2 - 2x + 1$.



Solution: As done in previous examples, we must distribute each term of our 1^{st} factor through our 2^{nd} factor. Since our 1^{st} factor is a trinomial, we must apply the Distribution of Multiplication Over Addition Property a total of 3 times.

As you can see with this example, Repeated Distribution starts to become overwhelming, prone for simple mistakes. Let's look at another method for polynomial by polynomial multiplication.

The Box Method

The *Box Method* provides a more organized layout for Repeated Distribution. This allows us to visualize which terms we must multiply together. To demonstrate the Box Method, let us take a second look at our examples.

Example 4:

Multiply x + 2 by $x^2 + 2x$.

	x^2	2x			x^2	2x	
x			$ \Longrightarrow$	x	x^3	$2x^2$	$\implies x^3 + 2x^2 + 2x^2 + 4x \implies x^3 + 4x^2 + 4x$
2				2	$2x^2$	4x	

Solution: First, we set up a grid for binomial by binomial multiplication. Now, we label our rows according to the terms within our 1^{st} factor and our columns according to the terms within our 2^{nd} factor. Next, we multiply each row by each column. Lastly, we add our products together and combine like terms.

Example 5:

Multiply x - 2 by $x^2 - 2x + 1$.

Solution: First, we set up a grid for binomial by trinomial multiplication. Now, we label our rows according to the terms within our 1^{st} factor and our columns according to the terms within our 2^{nd} factor. Next, we multiply each row by each column. Lastly, we add our products together and combine like terms.

Example 6:

Multiply $x^2 - 4x + 2$ by $x^2 - 2x + 1$.

	x^2	-2x	1			x^2	-2x	1	
x^2				\Rightarrow	x^2	x^4	$-2x^{3}$	x^2	\Rightarrow
-4x					-4x	$-4x^3$	$8x^2$	-4x	
2					2	$2x^2$	-4x	2	

 $x^{4} - 2x^{3} + x^{2} - 4x^{3} + 8x^{2} - 4x + 2x^{2} - 4x + 2 \Longrightarrow x^{4} - 6x^{3} + 11x^{2} - 8x + 2x^{2} - 4x + 2x^{2} - 4x^{2} - 4x^$

Solution: First, we set up a grid for trinomial by trinomial multiplication. Now, we label our rows according to the terms within our 1^{st} factor and our columns according to the terms within our 2^{nd} factor. Next, we multiply each row by each column. Lastly, we add our products together and combine like terms.