2.5 – A Polynomial Multiplied by a Polynomial

Below is an algebraic expression that prompts polynomial by polynomial multiplication:

$$(x + 2) \cdot (x^2 + 2x)$$

If your current thoughts are: "how the *heck* am I going to do this?!," I promise you that this section is no more difficult than the previous. In fact, it is pretty much the same – and with a new trick, you might actually find it easier!

Repeated Distribution

Repeated Distribution is a textbook approach for carrying out polynomial by polynomial multiplication. This method suggests that we repeat the Distributive Property of Multiplication Over Addition for each term of our 1^{st} factor.

Example 1:

Multiply $x + 2$ by $x^2 + 2x$.

$$(\boldsymbol{x} \mathbf{+ 2})(x^2 + 2x)$$

$$\boldsymbol{x}(x^2 + 2x) \mathbf{+ 2}(x^2 + 2x)$$

$$\Downarrow$$

$$(\boldsymbol{x} \cdot x^2 + \boldsymbol{x} \cdot 2x) \mathbf{+} (\mathbf{2} \cdot x^2 + \mathbf{2} \cdot 2x)$$

$$\Downarrow$$

$$x^3 + 2x^2 + 2x^2 + 4x$$

$$\Downarrow$$

$$x^3 + 4x^2 + 4x$$

Solution: First, we enclose both binomials in parentheses. Now, we distribute the 1^{st} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow x(x^2 + 2x)$, and we distribute the 2^{nd} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow 2(x^2 + 2x)$. Once distributed, we combine like terms.

Example 2:

Multiply $x - 2$ by $x^2 - 2x + 1$.

$$(\boldsymbol{x} \boldsymbol{-} \boldsymbol{2})(x^2 - 2x + 1)$$

$$\boldsymbol{x}(x^2 - 2x + 1) \boldsymbol{-} \boldsymbol{2}(x^2 - 2x + 1)$$

$$\Downarrow$$

$$(\boldsymbol{x} \cdot x^2 + \boldsymbol{x} \cdot -2x + \boldsymbol{x} \cdot 1) \boldsymbol{+} (\boldsymbol{-2} \cdot x^2 + \boldsymbol{-2} \cdot -2x + \boldsymbol{-2} \cdot 1)$$

$$\Downarrow$$

$$x^3 - 2x^2 + x - 2x^2 + 4x - 2$$

$$\Downarrow$$

$$x^3 - 4x^2 + 5x - 2$$

Solution: First, we enclose both binomials in parentheses. Now, we distribute the 1^{st} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow x(x^2 - 2x + 1)$, and we distribute the 2^{nd} term of our 1^{st} factor through our 2^{nd} factor $\rightarrow -2(x^2 - 2x + 1)$. Note, we must not ignore the subtraction sign. The subtraction sign is included as a negative sign in our 2^{nd} term. When distributing, be sure to distribute the entire term. Once distributed, we combine like terms.

Example 3:

Multiply $x^2 - 4x + 2$ by $x^2 - 2x + 1$.

$$(\boldsymbol{x^2} \boldsymbol{-} \boldsymbol{4x} \boldsymbol{+} \boldsymbol{2})(x^2 - 2x + 1)$$

$$\boldsymbol{x^2}(x^2 - 2x + 1) \boldsymbol{-} \boldsymbol{4x}(x^2 - 2x + 1) \boldsymbol{+} \boldsymbol{2}(x^2 - 2x + 1)$$

Solution: As done in previous examples, we must distribute each term of our 1^{st} factor through our 2^{nd} factor. Since our 1^{st} factor is a trinomial, we must apply the Distribution of Multiplication Over Addition Property a total of 3 times.

As you can see with this example, Repeated Distribution starts to become overwhelming, prone for simple mistakes. Let's look at another method for polynomial by polynomial multiplication.

The Box Method

The *Box Method* provides a more organized layout for Repeated Distribution. This allows us to visualize which terms we must multiply together. To demonstrate the Box Method, let us take a second look at our examples.

Example 4:

Multiply $x + 2$ by $x^2 + 2x$.

	x^2	$2x$
x		
2		

$\Longrightarrow$

	x^2	$2x$
x	$\boldsymbol{x^3}$	$\boldsymbol{2x^2}$
2	$\boldsymbol{2x^2}$	$\boldsymbol{4x}$

$\Longrightarrow x^3+2x^2+2x^2+4x \Longrightarrow x^3+4x^2+4x$

Solution: First, we set up a grid for binomial by binomial multiplication. Now, we label our rows according to the terms within our 1^{st} factor and our columns according to the terms within our 2^{nd} factor. Next, we multiply each row by each column. Lastly, we add our products together and combine like terms.

Example 5:

Multiply $x - 2$ by $x^2 - 2x + 1$.

	x^2	$-2x$	1
x			
-2			

$\Longrightarrow$

	x^2	$-2x$	1
x	$\boldsymbol{x^3}$	$\boldsymbol{-2x^2}$	$\boldsymbol{x}$
-2	$\boldsymbol{-2x^2}$	$\boldsymbol{4x}$	**-2**

$\Longrightarrow$

$$x^3 - 2x^2 + x - 2x^2 + 4x - 2 \Longrightarrow x^3 - 4x^2 + 5x - 2$$

Solution: First, we set up a grid for binomial by trinomial multiplication. Now, we label our rows according to the terms within our 1^{st} factor and our columns according to the terms within our 2^{nd} factor. Next, we multiply each row by each column. Lastly, we add our products together and combine like terms.

Example 6:

Multiply $x^2 - 4x + 2$ by $x^2 - 2x + 1$.

	x^2	$-2x$	1
x^2			
$-4x$			
2			

$\Longrightarrow$

	x^2	$-2x$	1
x^2	$\boldsymbol{x^4}$	$\boldsymbol{-2x^3}$	$\boldsymbol{x^2}$
$-4x$	$\boldsymbol{-4x^3}$	$\boldsymbol{8x^2}$	$\boldsymbol{-4x}$
2	$\boldsymbol{2x^2}$	$\boldsymbol{-4x}$	$\boldsymbol{2}$

$\Longrightarrow$

$$x^4 - 2x^3 + x^2 - 4x^3 + 8x^2 - 4x + 2x^2 - 4x + 2 \Longrightarrow x^4 - 6x^3 + 11x^2 - 8x + 2$$

Solution: First, we set up a grid for trinomial by trinomial multiplication. Now, we label our rows according to the terms within our 1^{st} factor and our columns according to the terms within our 2^{nd} factor. Next, we multiply each row by each column. Lastly, we add our products together and combine like terms.