CIRCULAR FORCES EQUATIONS

If the tangential acceleration is zero, the angular speed is constant.

horizontal circle

$$\sum F_r = T\cos(\phi) = \frac{m v^2}{R} \qquad \sum F_z = T\sin(\phi) - m g = 0 \qquad \sum F_{tan} = 0$$

Here φ is the angle in the vertical direction. This is assumed to be negligible (zero), if the mass is small.

vertical circle, constant speed

$$\text{TOP} \quad \sum F_r = F_N\,(\text{or T}) + m g = \frac{m v^2}{R} \qquad \sum F_z = 0 \qquad \sum F_{tan} = 0$$

$$\text{BOT} \quad \sum F_r = F_N\,(\text{or T}) - m g = \frac{m v^2}{R} \qquad \sum F_z = 0 \qquad \sum F_{tan} = 0$$

vertical circle, nonconstant speed (gravitational acceleration)

$$\sum F_r = F_N\,(\text{or T}) - mg\cos(\theta_t) = \frac{m v_t^2}{R} \qquad \sum F_z = 0 \qquad \sum F_{tan} = mg\sin(\theta_t) = m\,a_{tan}(t)$$

Here θ is the angle of the position vector, measured from the vertical. The time- (and angle-) dependent tangential acceleration leads to a nonlinear second-order differential equation for the angular position as a function of time, the solution to which is complicated. Thus in these problems the angle and velocity will be provided. The mathematics of this case is the same as for a pendulum.

round room

$$\sum F_r = F_N = \frac{m v^2}{R} \qquad \sum F_z = \mu F_N - m g = 0 \qquad \sum F_{tan} = 0$$

conical pendulum

$$\sum F_r = T\sin(\theta) = \frac{m v^2}{R} \qquad \sum F_z = T\cos(\theta) - m g = 0 \qquad \sum F_{tan} = 0$$

The angle θ is measured from the vertical.

curve, no banking

$$\sum F_r = \mu F_N = \frac{m v^2}{R} \qquad \sum F_z = F_N - m g = 0 \qquad \sum F_{tan} = 0$$

curve, banked (no friction)

$$\sum F_r = F_N\sin(\theta) = \frac{m v^2}{R} \qquad \sum F_z = F_N\cos(\theta) - m g = 0 \qquad \sum F_{tan} = 0$$