6 Trigonometric Functions: Right Triangle Approach

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6.1 Angle Measure

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Objectives

- Angle Measure
- Angles in Standard Position

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Angle Measure

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Angle Measure

An **angle** *AOB* consists of two rays R_1 and R_2 with a common vertex *O* (see Figure 1).

We often interpret an angle as a rotation of the ray R_1 onto R_2.

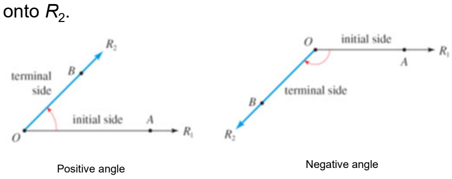

Figure 1

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Angle Measure

In this case R_1 is called the **initial side**, and R_2 is called the **terminal side** of the angle.

If the rotation is counterclockwise, the angle is considered **positive**, and if the rotation is clockwise, the angle is considered **negative**.

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Angle Measure

The **measure** of an angle is the amount of rotation about the vertex required to move R_1 onto R_2.

Intuitively, this is how much the angle "opens." One unit of measurement for angles is the **degree**.

An angle of measure 1 degree is formed by rotating the initial side $\frac{1}{360}$ of a complete revolution.

In calculus and other branches of mathematics a more natural method of measuring angles is used: *radian measure*.

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Angle Measure

DEFINITION OF RADIAN MEASURE

If a circle of radius 1 is drawn with the vertex of an angle at its center, then the measure of this angle in **radians** (abbreviated **rad**) is the length of the arc that subtends the angle (see Figure 2).

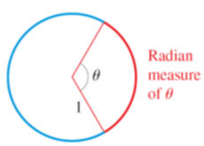

Figure 2

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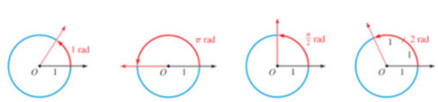
Angle Measure

The circumference of the circle of radius 1 is 2π, so a complete revolution has measure 2π rad, a straight angle has measure π rad, and a right angle has measure $\pi/2$ rad.

An angle that is subtended by an arc of length 2 along the unit circle has radian measure 2 (see Figure 3).



Radian measure
Figure 3

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Angle Measure

RELATIONSHIP BETWEEN DEGREES AND RADIANS

$$180° = \pi \text{ rad} \qquad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ} \qquad 1° = \frac{\pi}{180} \text{ rad}$$

1. To convert degrees to radians, multiply by $\frac{\pi}{180}$.
2. To convert radians to degrees, multiply by $\frac{180}{\pi}$.

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Example 1 – *Converting Between Radians and Degrees*

(a) Express 60° in radians. **(b)** Express $\frac{\pi}{6}$ rad in degrees.

Solution:
The relationship between degrees and radians gives

(a) $60° = 60\left(\frac{\pi}{180}\right) \text{ rad} = \frac{\pi}{3} \text{ rad}$

(b) $\frac{\pi}{6} \text{ rad} = \left(\frac{\pi}{6}\right)\left(\frac{180}{\pi}\right)$

$= 30°$

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Angle Measure

A note on terminology: We often use a phrase such as "a 30° angle" to mean *an angle whose measure is* 30°.

Also, for an angle θ we write $\theta = 30°$ or $\theta = \pi/6$ to mean *the measure of* θ *is* 30° *or* $\pi/6$ *rad*.

When no unit is given, the angle is assumed to be measured in radians.

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Angles in Standard Position

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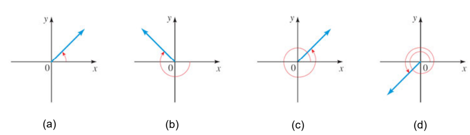
Angles in Standard Position

An angle is in **standard position** if it is drawn in the *xy*-plane with its vertex at the origin and its initial side on the positive *x*-axis.

Figure 5 gives examples of angles in standard position.



Angles in standard position
Figure 5

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Angles in Standard Position

Two angles in standard position are **coterminal** if their sides coincide.

In Figure 5 the angles in (a) and (c) are coterminal.

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Example 2 – *Coterminal Angles*

(a) Find angles that are coterminal with the angle $\theta = 30°$ in standard position.

(b) Find angles that are coterminal with the angle $\theta = \frac{\pi}{3}$ in standard position.

Solution:

(a) To find positive angles that are coterminal with θ, we add any multiple of 360°.

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Example 2 – *Solution*

cont'd

Thus

30° + 360° = 390° and 30° + 720° = 750°

are coterminal with $\theta = 30°$. To find negative angles that are coterminal with θ, we subtract any multiple of 360°.

Thus

30° – 360° = –330° and 30° – 720° = –690°

are coterminal with θ.

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Example 2 – *Solution*

cont'd

See Figure 6.

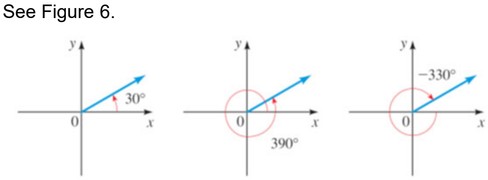

Figure 6

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Example 2 – *Solution*

cont'd

(b) To find positive angles that are coterminal with θ, we add any multiple of 2π.

Thus

$$\frac{\pi}{3} + 2\pi = \frac{7\pi}{3} \quad \text{and} \quad \frac{\pi}{3} + 4\pi = \frac{13\pi}{3}$$

are coterminal with $\theta = \pi/3$.

To find negative angles that are coterminal with θ, we subtract any multiple of 2π.

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Example 2 – *Solution*

cont'd

Thus

$$\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3} \quad \text{and} \quad \frac{\pi}{3} - 4\pi = -\frac{11\pi}{3}$$

are coterminal with θ. (See Figure 7.)

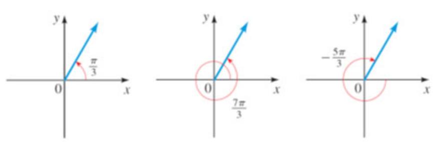

Figure 7

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